

**1**  $n$  is an integer greater than 1

Prove algebraically that  $n^2 - 2 - (n - 2)^2$  is always an even number.

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**(Total for Question 1 is 4 marks)**

2 Prove that the square of an odd number is always 1 more than a multiple of 4

**(Total for Question 2 is 4 marks)**

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**3** Given that  $n$  can be any integer such that  $n > 1$ , prove that  $n^2 - n$  is never an odd number.

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**(Total for Question 3 is 2 marks)**

- 4 Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

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**(Total for Question 4 is 3 marks)**

- 5 Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4

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**(Total for Question 5 is 3 marks)**

6 (a) Prove that

$$(2m + 1)^2 - (2n - 1)^2 = 4(m + n)(m - n + 1)$$

(3)

Sophia says that the result in part (a) shows that the difference of the squares of any two odd numbers must be a multiple of 4

(b) Is Sophia correct?

You must give reasons for your answer.

(1)

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**(Total for Question 6 is 4 marks)**

7  $n$  is an integer.

Prove algebraically that the sum of  $\frac{1}{2}n(n+1)$  and  $\frac{1}{2}(n+1)(n+2)$  is always a square number.

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**(Total for Question 7 is 2 marks)**

- 8 Prove algebraically that the straight line with equation  $x - 2y = 10$  is a tangent to the circle with equation  $x^2 + y^2 = 20$

(Total for Question 8 is 5 marks)