Answer ALL questions. Write your answers in the spaces provided.

1.

$$g(x) = \frac{2x+5}{x-3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Answer ALL questions. Write your answers in the spaces provided.

1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

5. $$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found. **(3)**

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point. **(3)**

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

6.

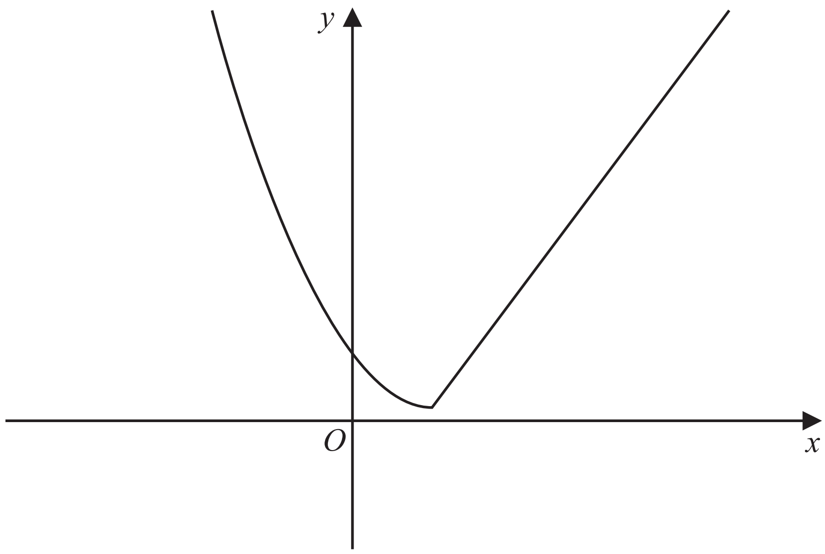

Figure 4

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $\mathrm{gg}(0)$.

(2)

(b) Find all values of x for which

$$\mathrm{g}(x) > 28$$

(4)

The function h is defined by

$$\mathrm{h}(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$\mathrm{h}^{-1}(x) = -\frac{1}{2}$$

(3)

4. The function f is defined by

$$f(x) = \frac{3x - 7}{x - 2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2)

(b) Show that $ff(x) = \dfrac{ax + b}{x - 3}$ where a and b are integers to be found.

(3)

7.

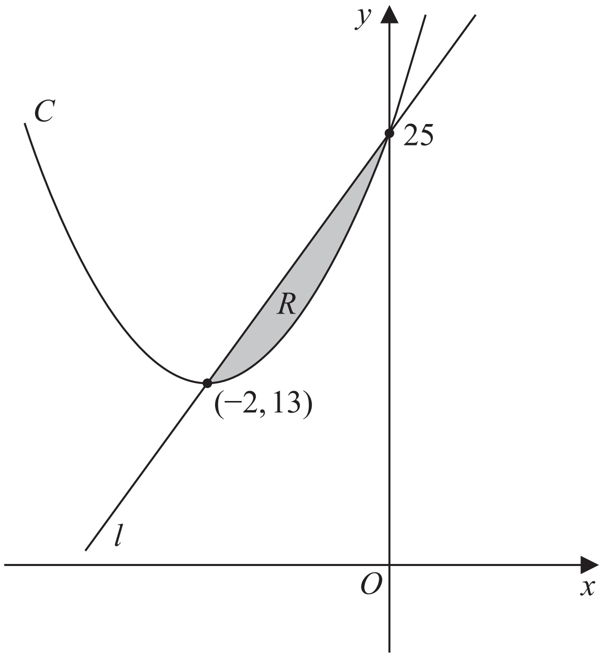

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ and a straight line l.

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $\mathrm{f}(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = \mathrm{f}(x)$

use inequalities to define R.

(5)

4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

(Total for Question 4 is 5 marks)

Leave blank

7. The function f is defined by

$$f: x \mapsto \frac{3x - 5}{x + 1}, \quad x \in \mathbb{R}, x \neq -1$$

(a) Find an expression for $f^{-1}(x)$

(3)

(b) Show that

$$ff(x) = \frac{x + a}{x - 1}, \quad x \in \mathbb{R}, x \neq -1, x \neq 1$$

where a is an integer to be determined.

(4)

The function g is defined by

$$g: x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, 0 \leqslant x \leqslant 5$$

(c) Find the value of fg(2)

(2)

(d) Find the range of g

(3)

Leave blank

4. Given that

$$f(x) = \frac{4}{3x+5}, \quad x > 0$$

$$g(x) = \frac{1}{x}, \qquad x > 0$$

(a) state the range of f,

(2)

(b) find $f^{-1}(x)$,

(3)

(c) find $fg(x)$.

(1)

(d) Show that the equation $fg(x) = gf(x)$ has no real solutions.

(4)

Leave blank

3. The function g is defined by

$$g(x) = \frac{6x}{2x + 3} \qquad x > 0$$

(a) Find the range of g.
(2)

(b) Find $g^{-1}(x)$ and state its domain.
(3)

(c) Find the function $gg(x)$, writing your answer as a single fraction in its simplest form.
(3)

Leave blank

3.

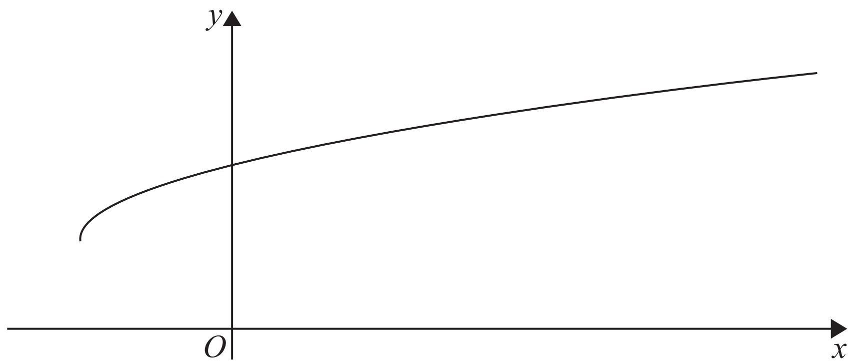

Figure 1

Figure 1 shows a sketch of part of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = 3 + \sqrt{x + 2}, \qquad x \geqslant -2$$

(a) State the range of g.

(1)

(b) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(3)

(c) Find the exact value of x for which

$$\mathrm{g}(x) = x$$

(4)

(d) Hence state the value of a for which

$$\mathrm{g}(a) = \mathrm{g}^{-1}(a)$$

(1)

Leave blank

1. The functions f and g are defined by

$$f : x \to 7x - 1, \qquad x \in \mathbb{R}$$

$$g : x \to \frac{4}{x-2}, \qquad x \neq 2, x \in \mathbb{R}$$

(a) Solve the equation $fg(x) = x$

(4)

(b) Hence, or otherwise, find the largest value of a such that $g(a) = f^{-1}(a)$

(1)

Leave blank

7.

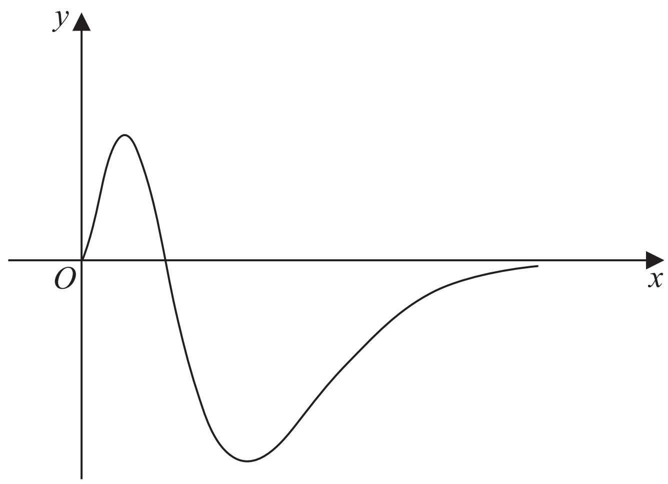

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \quad x \geqslant 0$$

(a) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(b) Hence find the range of g.

(6)

(c) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

Leave blank

6. The function f is defined by

$$f : x \rightarrow e^{2x} + k^2, \quad x \in \mathbb{R}, \quad k \text{ is a positive constant.}$$

(a) State the range of f.

(1)

(b) Find f^{-1} and state its domain.

(3)

The function g is defined by

$$g : x \rightarrow \ln(2x), \quad x > 0$$

(c) Solve the equation

$$g(x) + g(x^2) + g(x^3) = 6$$

giving your answer in its simplest form.

(4)

(d) Find fg(x), giving your answer in its simplest form.

(2)

(e) Find, in terms of the constant k, the solution of the equation

$$fg(x) = 2k^2$$

(2)

Leave blank

5.

$$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2 + x - 6}, \quad x > 3$$

(a) Show that $g(x) = \dfrac{x+1}{x-2}, \quad x > 3$ **(4)**

(b) Find the range of g. **(2)**

(c) Find the exact value of a for which $g(a) = g^{-1}(a)$. **(4)**

Leave blank

7. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from (–2, 10) to (2, 0) and from (2, 0) to (6, 4). A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

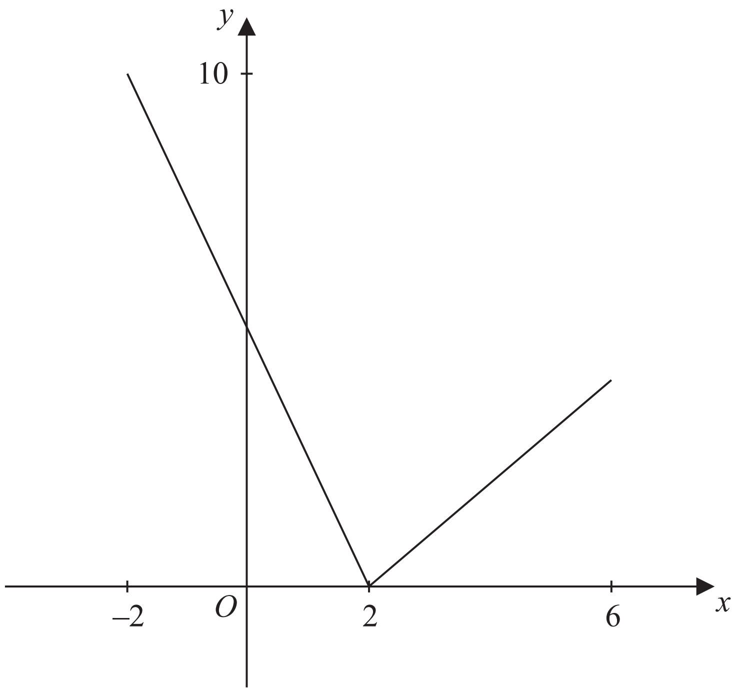

Figure 1

(a) Write down the range of f.
(1)

(b) Find ff(0).
(2)

The function g is defined by

$$\mathrm{g} : x \rightarrow \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $\mathrm{g}^{-1}(x)$
(3)

(d) Solve the equation $\mathrm{gf}(x) = 16$
(5)

11.

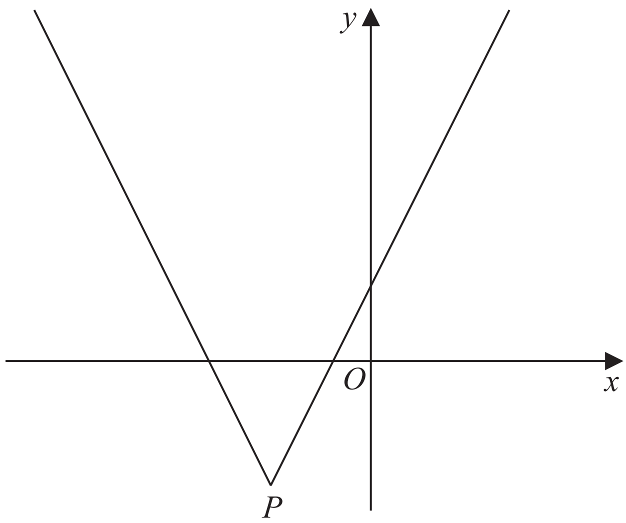

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

Leave blank

4.

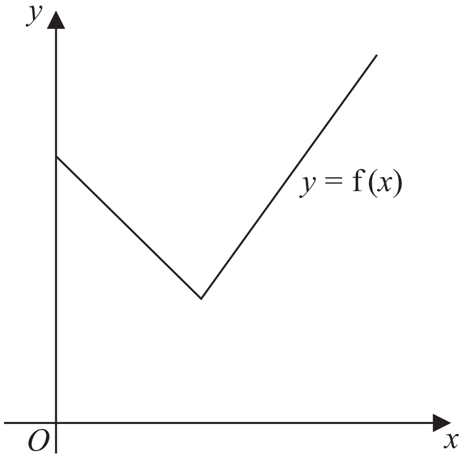

Figure 1

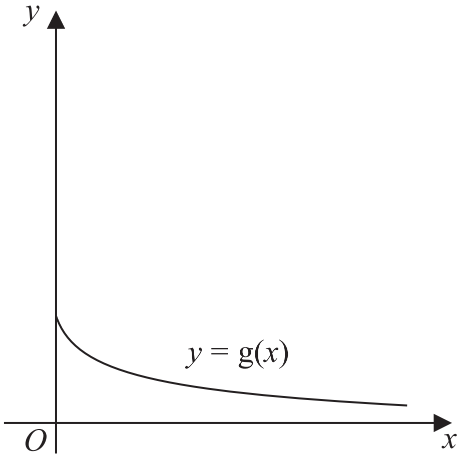

Figure 2

Figure 1 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

Figure 2 shows a sketch of part of the graph $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x + 9}{2x + 3}, \quad x \geqslant 0$$

(a) Find the value of fg(1)

(2)

(b) State the range of g

(2)

(c) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(4)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(d) state the range of possible values of k.

(3)

Leave blank

3. The function g is defined by

$$g : x \mapsto |8 - 2x|, \qquad x \in \mathbb{R}, \qquad x \geqslant 0$$

(a) Sketch the graph with equation $y = \text{g}(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(3)

(b) Solve the equation

$$|8 - 2x| = x + 5$$

(3)

The function f is defined by

$$f : x \mapsto x^2 - 3x + 1, \qquad x \in \mathbb{R}, \qquad 0 \leqslant x \leqslant 4$$

(c) Find fg(5).

(2)

(d) Find the range of f. You must make your method clear.

(4)

Leave blank

11.

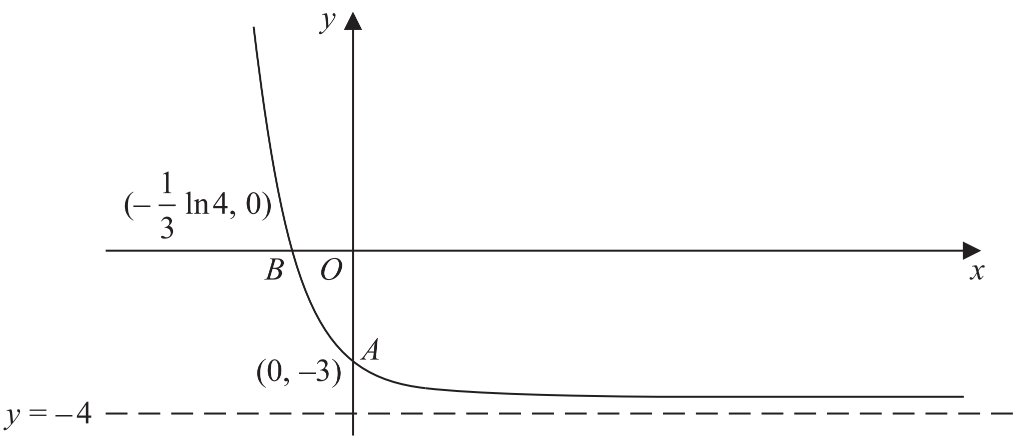

Figure 4

Figure 4 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

The curve meets the coordinate axes at the points $A\,(0, -3)$ and $B\,(-\frac{1}{3}\ln 4, 0)$ and the curve has an asymptote with equation $y = -4$

In separate diagrams, sketch the graph with equation

(a) $y = |\mathrm{f}(x)|$

(4)

(b) $y = 2\mathrm{f}(x) + 6$

(3)

On each sketch, give the exact coordinates of the points where the curve crosses or meets the coordinate axes and the equation of any asymptote.

Given that

$$\mathrm{f}(x) = \mathrm{e}^{-3x} - 4, \qquad x \in \mathbb{R}$$

$$\mathrm{g}(x) = \ln\left(\frac{1}{x+2}\right), \qquad x > -2$$

(c) state the range of f,

(1)

(d) find $\mathrm{f}^{-1}(x)$,

(3)

(e) express $\mathrm{fg}(x)$ as a polynomial in x.

(3)

Leave blank

6. Given that a and b are constants and that $a > b > 0$

(a) on separate diagrams, sketch the graph with equation

(i) $y = |x - a|$

(ii) $y = |x - a| - b$

Show on each sketch the coordinates of each point at which the graph crosses or meets the x-axis and the y-axis.

(5)

(b) Hence or otherwise find the complete set of values of x for which

$$|x - a| - b < \frac{1}{2}x$$

giving your answer in terms of a and b.

(4)

Leave blank

7.

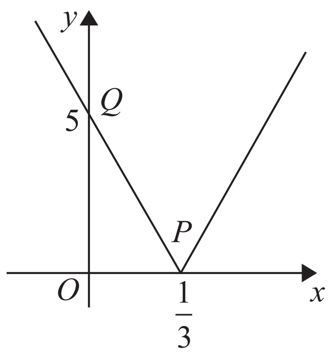

Figure 2

Figure 2 shows a sketch of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The point $P\left(\frac{1}{3}, 0\right)$ is the vertex of the graph.

The point $Q\,(0, 5)$ is the intercept with the y-axis.

Given that $\mathrm{f}(x) = |ax + b|$, where a and b are constants,

(a) (i) find all possible values for a and b,

(ii) hence find an equation for the graph.

(4)

(b) Sketch the graph with equation

$$y = \mathrm{f}\left(\frac{1}{2}x\right) + 3$$

showing the coordinates of its vertex and its intercept with the y-axis.

(3)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 4 8 3 2 5 A 0 2 0 4 8

Leave blank

9.

$$f(x) = 2\ln(x) - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(5)

(b) Find the exact solutions of the equation $|f(x)| = 4$

(4)

$$g(x) = e^{x+5} - 2, \quad x \in \mathbb{R}$$

(c) Find $gf(x)$, giving your answer in its simplest form.

(3)

(d) Hence, or otherwise, state the range of gf.

(1)

Leave blank

6. Given that a and b are positive constants,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$

(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(b) find c in terms of a.

(4)

Leave blank

2. Given that

$$f(x) = 2e^x - 5, \quad x \in \mathbb{R}$$

(a) sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(6)

(b) Deduce the set of values of x for which $f(x) = |f(x)|$

(1)

(c) Find the exact solutions of the equation $|f(x)| = 2$

(3)

Leave blank

5. (a) Sketch the graph with equation

$$y = |4x - 3|$$

stating the coordinates of any points where the graph cuts or meets the axes.

(2)

Find the complete set of values of x for which

(b)

$$|4x - 3| > 2 - 2x$$

(4)

(c)

$$|4x - 3| > \frac{3}{2} - 2x$$

(2)

Leave blank

4.

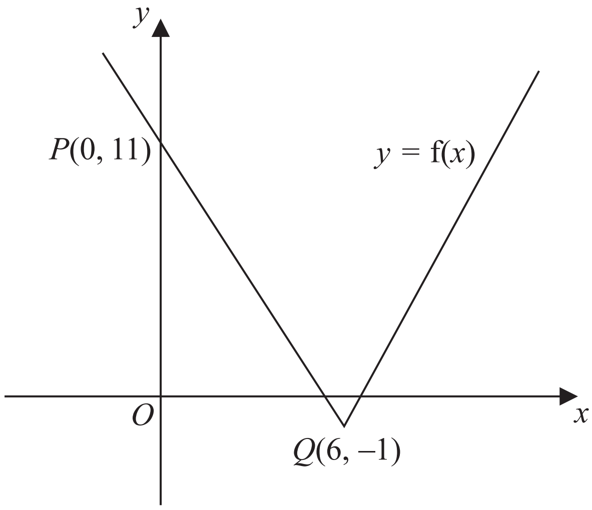

Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = 2\mathrm{f}(-x) + 3$

(3)

On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $\mathrm{f}(x) = a|x - b| - 1$, where a and b are constants,

(c) state the value of a and the value of b.

(2)

Leave blank

2.

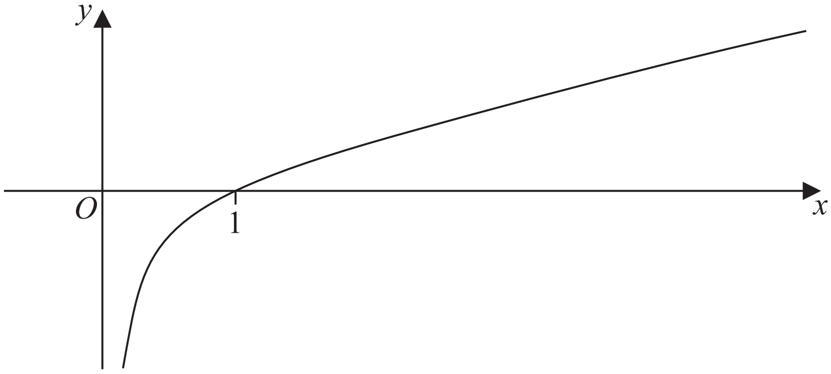

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$, $x > 0$, where f is an increasing function of x. The curve crosses the x-axis at the point (1, 0) and the line $x = 0$ is an asymptote to the curve.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$, $x > 0$

(2)

(b) $y = |f(x)|$, $x > 0$

(3)

Indicate clearly on each sketch the coordinates of the point at which the curve crosses or meets the x-axis.

Leave blank

4. The functions f and g are defined by

$$f : x \mapsto 2|x| + 3, \qquad x \in \mathbb{R},$$

$$g : x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(a) State the range of f.
(2)

(b) Find fg(1).
(2)

(c) Find g^{-1}, the inverse function of g.
(2)

(d) Solve the equation

$$gg(x) + [g(x)]^2 = 0$$

(5)

Leave blank

4.

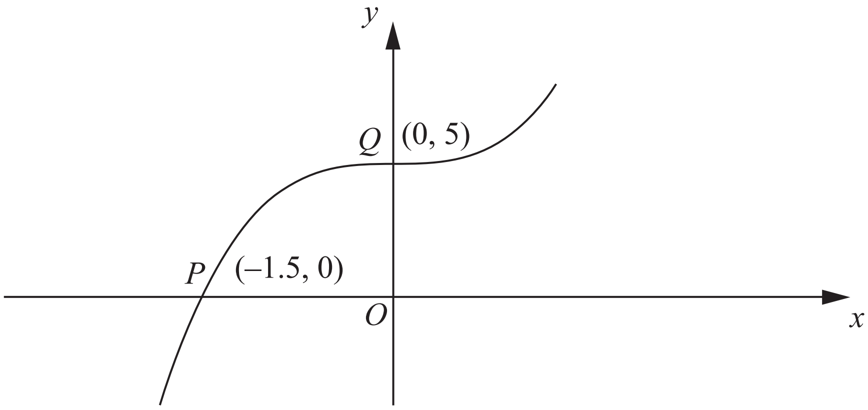

Figure 2

Figure 2 shows part of the curve with equation $y = \mathrm{f}(x)$
The curve passes through the points $P(-1.5,\ 0)$ and $Q(0,\ 5)$ as shown.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = \mathrm{f}(|x|)$

(2)

(c) $y = 2\mathrm{f}(3x)$

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Leave blank

3.

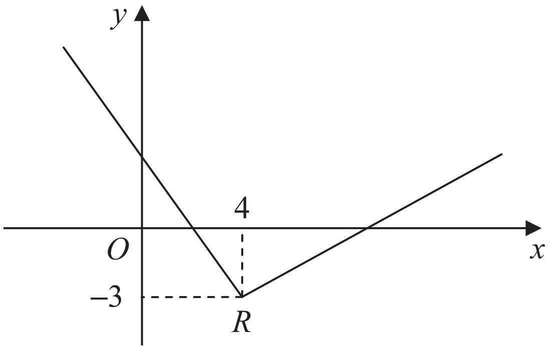

Figure 1

Figure 1 shows part of the graph of $y = \mathrm{f}(x),\ x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x+4)$,

(3)

(b) $y = |\mathrm{f}(-x)|$.

(3)

On each diagram, show the coordinates of the point corresponding to R.

Leave blank

2.

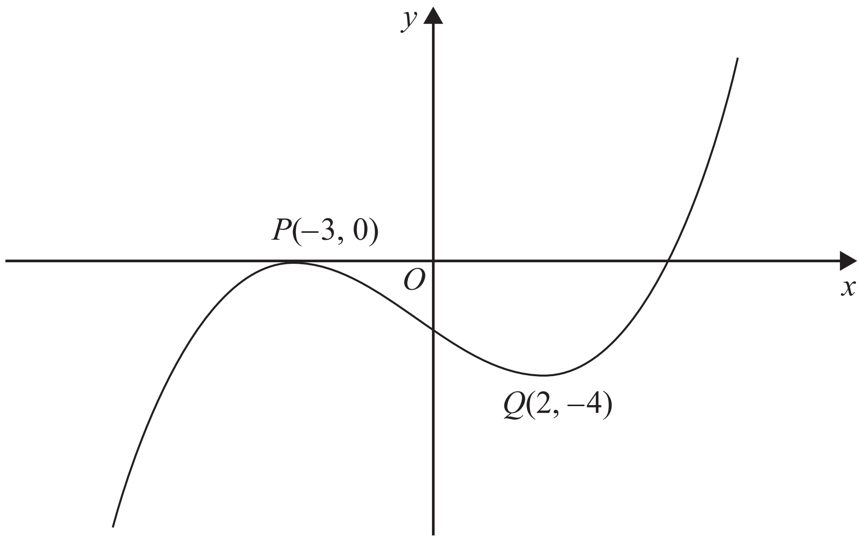

Figure 1

Figure 1 shows the graph of equation $y = f(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3f(x + 2)$

(3)

(b) $y = |f(x)|$

(3)

On each diagram, show the coordinates of any stationary points.