Question	Scheme	Marks	AOs
1	$g(x) = \frac{2x+5}{x-3},\ x \geq 5$		
(a) **Way 1**	$g(5) = \frac{2(5)+5}{5-3} = 7.5 \Rightarrow gg(5) = \frac{2("7.5")+5}{"7.5"-3}$	M1	1.1b
	$gg(5) = \frac{40}{9}\ \left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) **Way 2**	$gg(x) = \frac{2\left(\frac{2x+5}{x-3}\right)+5}{\left(\frac{2x+5}{x-3}\right)-3} \Rightarrow gg(5) = \frac{2\left(\frac{2(5)+5}{(5)-3}\right)+5}{\left(\frac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$gg(5) = \frac{40}{9}\ \left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	{Range:} $2 < y \leq \frac{15}{2}$	B1	1.1b
		(1)	
(c) **Way 1**	$y = \frac{2x+5}{x-3} \Rightarrow yx - 3y = 2x+5 \Rightarrow yx - 2x = 3y+5$	M1	1.1b
	$x(y-2) = 3y+5 \Rightarrow x = \frac{3y+5}{y-2}\ \left\{\text{or } y = \frac{3x+5}{x-2}\right\}$	M1	2.1
	$g^{-1}(x) = \frac{3x+5}{x-2},\quad 2 < x \leq \frac{15}{2}$	A1ft	2.5
		(3)	
(c) **Way 2**	$y = \frac{2x-6+11}{x-3} \Rightarrow y = 2 + \frac{11}{x-3} \Rightarrow y-2 = \frac{11}{x-3}$	M1	1.1b
	$x-3 = \frac{11}{y-2} \Rightarrow x = \frac{11}{y-2}+3\ \left\{\text{or } y = \frac{11}{x-2}+3\right\}$	M1	2.1
	$g^{-1}(x) = \frac{11}{x-2}+3,\quad 2 < x \leq \frac{15}{2}$	A1ft	2.5
		(3)	
		(6 marks)	

Notes for Question 1

(a)	
M1:	Full method of attempting $g(5)$ and substituting the result into g
Note:	**Way 2:** Attempts to substitute $x = 5$ into $\frac{2\left(\frac{2x+5}{x-3}\right)+5}{\left(\frac{2x+5}{x-3}\right)-3}$, o.e. Note that $gg(x) = \frac{9x-5}{14-x}$
A1:	Obtains $\frac{40}{9}$ or $4\frac{4}{9}$ or $4.\dot{4}$ or an exact equivalent
Note:	Give A0 for 4.4 or 4.444... without reference to $\frac{40}{9}$ or $4\frac{4}{9}$ or $4.\dot{4}$

Question	Scheme	Marks	AOs
1	Attempts $\mathrm{f}(-3)=3\times(-3)^3+2a\times(-3)^2-4\times-3+5a=0$	M1	3.1a
	Solves linear equation $23a=69\Rightarrow a=\ldots$	M1	1.1b
	$a=3$ cso	A1	1.1b
		(3)	
			(3 marks)

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $\mathrm{f}(-3)=0$ leading to a correct equation in a.
Missing brackets may be recovered.
Other methods may be seen but they are more demanding
If division is attempted must produce **a correct equation** in a similar way to the $\mathrm{f}(-3)=0$ method

$$\begin{array}{r l}
 & 3x^2+(2a-9)x+23-6a \\
x+3 \big) & 3x^3+2ax^2-4x+5a \\
 & \underline{3x^3+9x^2} \\
 & (2a-9)x^2-4x \\
 & \underline{(2a-9)x^2+(6a-27)x} \\
 & (23-6a)x+5a \\
 & (23-6a)x+69-18a
\end{array}$$

So accept $5a=69-18a$ or equivalent, where it implies that the remainder will be 0
You may also see variations on the table below. In this method the terms in x are equated to -4

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$6a-27+\frac{5a}{3}=-4$$

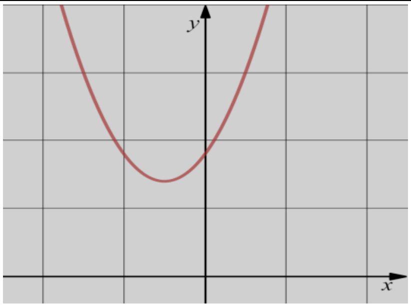
M1: This is scored for an attempt at solving a linear equation in a.
For the main scheme it is dependent upon having attempted $\mathrm{f}(\pm3)=0$. Allow for a linear equation in a leading to $a=\ldots$. Don't be too concerned with the mechanics of this.

Via division accept $x+3\overline{\big)3x^3+2ax^2-4x+5a}$ with quotient $3x^2\ldots$ followed by a remainder in a set $=0\Rightarrow a=\ldots$

or two terms in a are equated so that the remainder = 0
FYI the correct remainder via division is $23a+12-81$ oe

A1: $a=3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

Question	Scheme	Marks	AOs
5 (a)	$2x^2+4x+9=2(x\pm k)^2 \pm ....$ $\quad a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2 \pm ...$ $\quad a=2\ \&b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	A1	2.2a
	(ii) $\text{h}(x)=\dfrac{21}{"2(x+1)^2+7"} \Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<\text{h}(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2 \pm ....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2 \pm ...$ or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c \equiv ax^2+2abx+ab^2+c$

Question	Scheme	Marks	AOs
6 (a)	$\text{gg}(0) = \text{g}((0-2)^2+1) = \text{g}(5) = 4(5)-7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2+1=28 \Rightarrow x=...$ or $4x-7=28 \Rightarrow x=...$	M1	1.1b
	At least one critical value $x=2-3\sqrt{3}$ or $x=\frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2+1=28 \Rightarrow x=...$ and $4x-7=28 \Rightarrow x=...$	M1	1.1b
	Correct final answer of '$x<2-3\sqrt{3}$, $x>\frac{35}{4}$'	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2-3\sqrt{3}$ is accepted for any of the A marks	**(4)**	
(c)	<u>h is a one-one</u> {function (or mapping) so has an inverse} <u>g is a many-one</u> {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) Way 1	$\left\{\text{h}^{-1}(x) = -\frac{1}{2} \Rightarrow\right\}$ $x = \text{h}\left(-\frac{1}{2}\right)$	M1 B1 on epen	1.1b
	$x=\left(-\frac{1}{2}-2\right)^2+1$ **Note:** Condone $x=\left(\frac{1}{2}-2\right)^2+1$	M1	1.1b
	$\Rightarrow x=7.25$ only **cso**	A1	2.2a
		(3)	
(d) Way 2	{their $\text{h}^{-1}(x)$ } $= \pm 2 \pm \sqrt{x \pm 1}$	M1	1.1b
	Attempts to solve $\pm 2 \pm \sqrt{x\pm 1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x \pm 1} = ...$	M1	1.1b
	$\Rightarrow x=7.25$ only **cso**	A1	2.2a
		(3)	
		(10 marks)	

Notes for Question 6

(a)	
M1:	Uses a complete method to find $\text{gg}(0)$. E.g. • Substituting $x=0$ into $(0-2)^2+1$ and the result of this into the relevant part of $\text{g}(x)$ • Attempts to substitute $x=0$ into $4((x-2)^2+1)-7$ or $4(x-2)^2-3$
A1:	$\text{gg}(0)=13$
(b)	
M1:	See scheme
A1:	See scheme
M1:	See scheme
A1:	Brings all the strands of the problem together to give a correct solution.
Note:	You can ignore inequality symbols for any of the M marks
Note:	If a 3TQ is formed (e.g. $x^2-4x-23=0$) then a correct method for solving a 3TQ is required for the relevant method mark to be given.
Note:	Writing $(x-2)^2+1=28 \Rightarrow (x-2)+1=\sqrt{28} \Rightarrow x=-1+\sqrt{28}$ (i.e. taking the square-root of each term to solve $(x-2)^2+1=28$ is not considered to be an acceptable method)
Note:	Allow set notation. E.g. $\{x \in \mathbb{R}: x<2-3\sqrt{3} \cup x>8.75\}$ is fine for the final A mark

Question	Scheme	Marks	AOs
4 (a)	Either attempts $\frac{3x-7}{x-2}=7 \Rightarrow x=...$ Or attempts $f^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $ff(x)=\frac{3\times\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2}=\frac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\frac{2x-7}{x-3}$	A1	2.1
		(3)	
		(5 marks)	
Notes:			

(a)

M1: For either attempting to solve $\frac{3x-7}{x-2}=7$. Look for an attempt to multiply by the $(x-2)$ leading to a value for x.

Or score for substituting in $x=7$ in $f^{-1}(x)$. FYI $f^{-1}(x)=\frac{2x-7}{x-3}$

The method for finding $f^{-1}(x)$ should be sound, but you can condone slips.

A1: $\frac{7}{4}$

(b)

M1: For an attempt at fully substituting $\frac{3x-7}{x-2}$ into $f(x)$. Condone slips but the expression must have a correct form. E.g. $\frac{3\times\left(\frac{*\_*}{*\_*}\right)-a}{\left(\frac{*\_*}{*\_*}\right)-b}$ where a and b are positive constants.

dM1: Attempts to multiply **all** terms on the numerator and denominator by $(x-2)$ to create a fraction $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are linear expressions. Condone $\frac{P(x)}{Q(x)}\times\frac{x-2}{x-2}$

A1: Reaches $\frac{2x-7}{x-3}$ via careful and accurate work. Implied by $a=2, b=-7$ following correct work.

....

Methods involving $\frac{3x-7}{x-2}\equiv a+\frac{b}{x-2}$ may be seen. The scheme can be applied in a similar way

FYI $\frac{3x-7}{x-2}\equiv 3-\frac{1}{x-2}$

Question	Scheme	Marks	AOs
7	Attempts equation of line Eg Substitutes $(-2,13)$ into $y = mx + 25$ and finds m	M1	1.1b
	Equation of l is $y = 6x + 25$	A1	1.1b
	Attempts equation of C Eg Attempts to use the intercept $(0,25)$ within the equation $y = a(x \pm 2)^2 + 13$, in order to find a	M1	3.1a
	Equation of C is $y = 3(x+2)^2 + 13$ or $y = 3x^2 + 12x + 25$	A1	1.1b
	Region R is defined by $3(x+2)^2 + 13 < y < 6x + 25$ o.e.	B1ft	2.5
		(5)	
			(5 marks)

Notes:

The first two marks are awarded for finding the equation of the line

M1: Uses the information in an attempt to find an equation for the line l.

E.g. Attempt using two points: Finds $m = \pm\dfrac{25-13}{2}$ and uses of one of the points in their $y = mx + c$ or equivalent to find c. Alternatively uses the intercept as shown in main scheme.

A1: $y = 6x + 25$ seen or implied. This alone scores the first two marks. Do not accept $l = 6x + 25$

It must be in the form $y = ...$ but the correct equation can be implied from an inequality. E.g. $.... < y < 6x + 25$.

The next two marks are awarded for finding the equation of the curve

M1: A complete method to find the constant a in $y = a(x \pm 2)^2 + 13$ or the constants a, b in $y = ax^2 + bx + 25$.

An alternative to the main scheme is deducing equation is of the form $y = ax^2 + bx + 25$ and setting and solving a pair of simultaneous equations in a and b using the point $(-2, 13)$ the gradient being 0 at $x = -2$. Condone slips. Implied by $C = 3x^2 + 12x + 25$ or $3x^2 + 12x + 25$

FYI the correct equations are $13 = 4a - 2b + 25$ $(2a - b = -6)$ and $-4a + b = 0$

A1: $y = 3(x+2)^2 + 13$ or equivalent such as $y = 3x^2 + 12x + 25$, $\mathrm{f}(x) = 3(x+2)^2 + 13$.

Do not accept $C = 3x^2 + 12x + 25$ or just $3x^2 + 12x + 25$ for the A1 but may be implied from an inequality or from an attempt at the area, E.g. $\int 3x^2 + 12x + 25 \, \mathrm{d}x$

B1ft: Fully defines the region R. Follow through on their equations for l and C.

Allow strict or non -strict inequalities as long as they are used consistently.

E.g. Allow for example "$3(x+2)^2 + 13 < y < 6x + 25$" "$3(x+2)^2 + 13 \leqslant y \leqslant 6x + 25$"

Allow the inequalities to be given separately, e.g. $y < 6x + 25$, $y > 3(x+2)^2 + 13$. Set notation may be used so $\{(x,y) : y > 3(x+2)^2 + 13\} \cap \{(x,y) : y < 6x + 25\}$ is fine but condone with or without any of $(x,y) \leftrightarrow y \leftrightarrow x$

Incorrect examples include "$y < 6x + 25$ **or** $y > 3(x+2)^2 + 13$", $\{(x,y) : y > 3(x+2)^2 + 13\} \cup \{(x,y) : y < 6x + 25\}$

Values of x could be included but they must be correct. So $3(x+2)^2 + 13 < y < 6x + 25, x < 0$ is fine

If there are multiple solutions mark the final one.

Question	Scheme	Marks	AOs
4 (a)	$\text{gf}(x) = 3\ln e^x$	M1	1.1b
	$= 3x, \ (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$\text{gf}(x) = \text{fg}(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln e^x$

An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $e^{3\ln x}$ = their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$

or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

Question Number	Scheme (Method 1)	Scheme (Method 2)	Marks
7. (a)	Method 1	Method 2	
	$y = \dfrac{3x-5}{x+1}$	$y = 3 - \dfrac{8}{x+1}$	
	$y(x+1) = 3x - 5 \Rightarrow xy + y = 3x - 5$	$\dfrac{8}{x+1} = 3 - y$ so $x + 1 = \dfrac{8}{3-y}$	M1
	$y + 5 = 3x - xy \Rightarrow y + 5 = x(3 - y)$ $\Rightarrow \dfrac{y+5}{3-y} = x$	$x = \dfrac{8}{3-y} - 1$	M1
	Hence $\left(\mathrm{f}^{-1}(x)\right) = \dfrac{x+5}{3-x}$ $\quad (x \in \mathbb{R},\ x \neq 3)$	Hence $\left(\mathrm{f}^{-1}(x)\right) = \dfrac{8}{3-x} - 1$ $\quad (x \in \mathbb{R},\ x \neq 3)$	A1 oe
			[3]
(b)	$\mathrm{ff}(x) = \dfrac{3\left(\dfrac{3x-5}{x+1}\right) - 5}{\left(\dfrac{3x-5}{x+1}\right) + 1}$	$\mathrm{ff}(x) = 3 - \dfrac{8}{3 - \dfrac{8}{x+1} + 1}$	M1 A1
	$= \dfrac{\dfrac{3(3x-5) - 5(x+1)}{x+1}}{\dfrac{(3x-5) + (x+1)}{x+1}}$	$\mathrm{ff}(x) = 3 - \dfrac{8(x+1)}{4x-4}$	M1
	$= \dfrac{9x - 15 - 5x - 5}{3x - 5 + x + 1} = \dfrac{4x - 20}{4x - 4}$		
	$= \dfrac{x-5}{x-1}$ (note that $a = -5$.)	$= \dfrac{x-5}{x-1}$	A1
			[4]
(c)	$\mathrm{fg}(2) = \mathrm{f}(4-6) = \mathrm{f}(-2) = \dfrac{3(-2) - \text{"5"}}{-2+1}$ $;= 11$ or substitute 2 into $\mathrm{fg}(x) = \dfrac{3(x^2 - 3x) - 5}{x^2 - 3x + 1}$ $;= 11$		M1; A1
			[2]
(d)	$\mathrm{g}(x) = x^2 - 3x = (x - 1.5)^2 - 2.25$. Hence $\mathrm{g}_{\min} = -2.25$		M1
	Either $\mathrm{g}_{\min} = -2.25$ or $\mathrm{g}(x) \geqslant -2.25$ $\quad$ or $\mathrm{g}(5) = 25 - 15 = 10$		B1
	$\underline{-2.25 \leqslant \mathrm{g}(x) \leqslant 10}$ or $\underline{-2.25 \leqslant y \leqslant 10}$		A1
			[3] **12**

Qu	Scheme	Marks
4.(a)	$0 < \mathrm{f}(x) < \frac{4}{5}$	M1A1 **(2)**
(b)	$y = \dfrac{4}{3x+5} \quad \Rightarrow (3x+5)y = 4$	M1
	$\Rightarrow x = \dfrac{4-5y}{3y}$	dM1
	$\mathrm{f}^{-1}(x) = \dfrac{4-5x}{3x} \qquad \left(0 < x < \frac{4}{5}\right)$	A1o.e. **(3)**
(c)	$\mathrm{fg}(x) = \dfrac{4}{\frac{3}{x}+5}$	B1 (1)
(d)	$\dfrac{3x+5}{4} = \dfrac{4}{\frac{3}{x}+5}$	M1
	$15x^2 + 18x + 15 = 0$	A1
	Uses $18^2 < 4\times15\times15$ and so deduce no real roots	M1 A1 **(4)** **(10 marks)**

(a)
M1: One limit such as $y > 0$ or $y < 0.8$. Condone for this mark both limits but with x (not y) or with the boundary included. For example $[0,0.8], 0 < x < 0.8, 0 \leqslant y \leqslant 0.8$

A1: Fully correct so accept $0 < \mathrm{f}(x) < \frac{4}{5}$ and exact equivalents $0 < y < \frac{4}{5}$ $(0, 0.8)$

(b)
M1: Set $y = \mathrm{f}(x)$ or $x = \mathrm{f}(y)$ and multiply both sides by denominator.
dM1:Make x (or a swapped y) the subject of the formula. Condone arithmetic slips

A1: o.e for example $y/\mathrm{f}^{-1}(x) = \frac{1}{3}\left(\frac{4}{x} - 5\right)$ or $y = \left(\frac{4}{x} - 5\right)\Big/3$ - do not need domain for this mark. ISW after a correct answer.

(c) Mark parts c and d together

B1: $\mathrm{fg}(x) = \dfrac{4}{\frac{3}{x}+5}$ - allow any correct form then isw

(d)
M1: Sets $\mathrm{fg}(x) = \mathrm{gf}(x)$ with **both sides correct** (but may be unsimplified) and forms a quadratic in x. Do not withhold this mark if fg or gf was originally correct but was "simplified" incorrectly and set equal to a correct gf
A1: Correct 3TQ. It need not be all on one side of the equation. The =0 can be implied by later work
M1: Attempts the discriminant or attempts the formula or attempts to complete the square.
A1: Completely correct work (cso) and conclusion. If $b^2 - 4ac$ has been found it must be correct (-576)

Question Number	Scheme	Marks
3 (a)	$0<\mathrm{g}<3$	M1A1
		(2)
(b)	$y=\dfrac{6x}{2x+3}\Rightarrow 2xy+3y=6x\Rightarrow(6-2y)x=3y\Rightarrow x=\dfrac{3y}{(6-2y)}$	M1A1
	$\Rightarrow \mathrm{g}^{-1}(x)=\dfrac{3x}{(6-2x)} \qquad 0<x<3$	A1ft
		(3)
(c)	$\mathrm{gg}(x)=\mathrm{g}\left(\dfrac{6x}{2x+3}\right)=\dfrac{6\times\dfrac{6x}{2x+3}}{2\times\dfrac{6x}{2x+3}+3}$	M1
	$=\dfrac{6\times 6x}{2\times 6x+3(2x+3)}$	dM1
	$=\dfrac{36x}{18x+9}=\dfrac{4x}{2x+1}$	A1
		(3)
		(8 marks)

(a)

M1: For one 'end' fully correct $\mathrm{g}(x)>0$ (**not** $x>0$) or $\mathrm{g}(x)<3$ (**not** $x<3$) or both ends (incorrect) eg. accept $0\leqslant \mathrm{g}\leqslant 3$. Accept incorrect notation such as $0<x<3$ for this mark but **not** $x>0$ or $x<3$ **on their own**.
Allow use of f rather than g for the M mark but not the A mark.

A1: Accept $0<\mathrm{g}<3$, $0<y<3$, $\mathrm{g}(x)>0$ and $\mathrm{g}(x)<3$, $(0,3)$

(b)

M1: An attempt to make x or a replaced y the subject of the formula. The minimum expectation is that there is an attempt to cross multiply, expand and collect/factorise terms in x or a replaced y and obtain $x=\dfrac{\pm 3y}{(\pm 6\pm 2y)}$ or equivalent i.e. sign errors only on their algebra.

A1: $x=\dfrac{3y}{(6-2y)}$ or $\dfrac{-3y}{(2y-6)}$ or $y=\dfrac{3x}{(6-2x)}$ or $\dfrac{-3x}{(2x-6)}$ or $-\dfrac{3}{2}-\dfrac{9}{2(x-3)}$ etc. Allow $2(x-3)$ for $(2x-6)$.

A1ft: $\mathrm{g}^{-1}(x)=\dfrac{3x}{(6-2x)}\left(\text{or}\dfrac{-3x}{(2x-6)}\right)$ **and** $0<x<3$. You can follow through on any range from part (a) but the domain must be in terms of x not in terms of e.g. $\mathrm{g}(x)$ or $\mathrm{g}^{-1}(x)$. Do not allow $x\in\mathbb{R}$
Accept $y=\dfrac{3x}{(6-2x)}\left(\text{or}\dfrac{-3x}{(2x-6)}\right) \qquad 0<x<3$. Allow $2(x-3)$ for $(2x-6)$.

(c)

M1: Attempts to find $\mathrm{gg}(x)$ by finding $\mathrm{g}\left(\dfrac{6x}{2x+3}\right)$

dM1: Correct processing to obtain a single fraction of the form $\dfrac{a}{b}$. Achieved by,

- multiplying both numerator and denominator by $(2x+3)$ (must multiply both terms in the denominator)

Question Number	Scheme	Marks
3.(a)	$y \geqslant 3$	B1
		(1)
(b)	$y = 3+\sqrt{x+2} \Rightarrow y-3=\sqrt{x+2} \Rightarrow x=(y-3)^2-2$	M1 A1
	$\Rightarrow g^{-1}(x)=(x-3)^2-2,\quad \text{with } x \geqslant 3$	A1
		(3)
(c)	$g(x)=x \Rightarrow 3+\sqrt{x+2}=x$	
	$\Rightarrow x+2=(x-3)^2 \Rightarrow x^2-7x+7=0$	M1, A1
	$\Rightarrow x=\dfrac{7\pm\sqrt{21}}{2} \Rightarrow x=\dfrac{7+\sqrt{21}}{2}$ only	M1, A1
		(4)
(d)	$a=\dfrac{7+\sqrt{21}}{2}$	B1 ft
		(1)
		9 marks
(c) Alt	Solves $g^{-1}(x)=x \Rightarrow (x-3)^2-2=x$	
	$\Rightarrow x^2-7x+7=0$	M1, A1
	$\Rightarrow x=\dfrac{7\pm\sqrt{21}}{2} \Rightarrow x=\dfrac{7+\sqrt{21}}{2}$ only	dM1, A1
		(4)

(a)

B1 States the correct range for g Accept $g(x) \geqslant 3, g \geqslant 3$, Range $\geqslant 3, [3,\infty)$ Range is greater than or equal to 3
Condone $f \geqslant 3$ Do not accept $g(x)>3, x \geqslant 3, (3,\infty)$

(b)

M1 Attempts to make x or a swapped y the subject of the formula. The minimum expectation is that the 3 is moved over followed by an attempt to square both sides. Condone for this mark $\sqrt{x+2}=y\pm3 \Rightarrow x+2=y^2\pm9$

A1 Achieves $x=(y-3)^2-2$ or if swapped $y=(x-3)^2-2$ or equivalent such as $x=y^2-6y+7$

A1 Requires a correct function in x + correct domain **or** a correct function in x with a correct follow through on the range in (a) but do not follow through on $x \in \mathbb{R}$

Question	Scheme		Marks
1(a)	$\text{fg}(x)=\dfrac{28}{x-2}-1$	$\left(=\dfrac{30-x}{x-2}\right)$	M1
	Sets $\text{fg}(x)=x \Rightarrow \dfrac{28}{x-2}-1=x$ $\Rightarrow 28=(x+1)(x-2)$ $\Rightarrow x^2-x-30=0$ $\Rightarrow (x-6)(x+5)=0$ $\Rightarrow x=6\,,\ x=-5$		M1 dM1 A1 **(4)**
(b)	$a=6$		B1 ft **(1)** **5 marks**
Alt 1(a)	$\text{fg}(x)=x \Rightarrow \text{g}(x)=\text{f}^{-1}(x)$ $\dfrac{4}{x-2}=\dfrac{x+1}{7}$ $\Rightarrow x^2-x-30=0$ $\Rightarrow (x-6)(x+5)=0$ $\Rightarrow x=6\,,\ x=-5$		M1 M1 dM1 A1 **4 marks**
S. Case	Uses $\text{gf}(x)$ instead $\text{fg}(x)$ $\dfrac{4}{7x-1-2}=x$ $\Rightarrow 7x^2-3x-4=0$ $\Rightarrow (7x+4)(x-1)=0$ $\Rightarrow x=-\dfrac{4}{7},\quad x=1$	Makes an error on $\text{fg}(x)$ Sets $\text{fg}(x)=x \Rightarrow \dfrac{7\times 4}{7\times(x-2)}-1=x$ $\Rightarrow x^2-x-6=0$ $\Rightarrow (x+2)(x-3)=0$ $\Rightarrow x=-2,\quad x=3$	M0 M1 dM1 A0 **2 out of 4 marks**

(a)

M1 Sets or implies that $\text{fg}(x)=\dfrac{28}{x-2}-1$ Eg accept $\text{fg}(x)=7\left(\dfrac{4}{x-2}\right)-1$ followed by $\text{fg}(x)=\dfrac{7\times 4}{x-2}-1$

Alternatively sets $\text{g}(x)=\text{f}^{-1}(x)$ where $\text{f}^{-1}(x)=\dfrac{x\pm 1}{7}$

Note that $\text{fg}(x)=7\left(\dfrac{4}{x-2}\right)-1=\dfrac{28}{7(x-2)}-1$ is M0

M1 Sets up a 3TQ (= 0) from an attempt at $\text{fg}(x)=x$ or $\text{g}(x)=\text{f}^{-1}(x)$

dM1 Method of solving 3TQ (= 0) to find at least one value for x. See "General Priciples for Core Mathematics" on page 3 for the award of the mark for solving quadratic equations
This is dependent upon the previous M. You may just see the answers following the 3TQ.

A1 Both $x=6$ and $x=-5$

(b)

B1ft For $a=6$ but you may follow through on the largest solution from part (a) provided more than one answer was found in (a). Accept 6, $a=6$ and even $x=6$
Do not award marks for part (a) for work in part (b).

Question Number	Scheme	Marks
7.(a)	Applies $vu'+uv'$ to $\left(x^2-x^3\right)\mathrm{e}^{-2x}$	
	$\mathrm{g}'(x)=\left(x^2-x^3\right)\times -2\mathrm{e}^{-2x}+\left(2x-3x^2\right)\times \mathrm{e}^{-2x}$	M1 A1
	$\mathrm{g}'(x)=\left(2x^3-5x^2+2x\right)\mathrm{e}^{-2x}$	A1
		(3)
(b)	Sets $\left(2x^3-5x^2+2x\right)\mathrm{e}^{-2x}=0 \Rightarrow 2x^3-5x^2+2x=0$	M1
	$x\left(2x^2-5x+2\right)=0 \Rightarrow x=(0),\frac{1}{2},2$	M1,A1
	Sub $x=\frac{1}{2},2$ into $g(x)=\left(x^2-x^3\right)\mathrm{e}^{-2x} \Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8\mathrm{e}},\quad g(2)=-\frac{4}{\mathrm{e}^4}$	dM1,A1
	Range $-\frac{4}{\mathrm{e}^4}\leqslant g(x)\leqslant\frac{1}{8\mathrm{e}}$	A1 **(6)**
(c)	Accept $\mathrm{g}(x)$ is NOT a ONE to ONE function	
	Accept $\mathrm{g}(x)$ is a MANY to ONE function	B1
	Accept $\mathrm{g}^{-1}(x)$ would be ONE to MANY	**(1)**
		(10 marks)

Note that parts (a) and (b) can be scored together. Eg accept work in part (b) for part (a)

(a)

M1 **Uses the product rule** $vu'+uv'$ with $u=x^2-x^3$ and $v=e^{-2x}$ or vice versa. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $vu'+uv'$. If the rule is not quoted nor implied only accept expressions of the form $\left(x^2-x^3\right)\times\pm A\mathrm{e}^{-2x}+\left(Bx\pm Cx^2\right)\times\mathrm{e}^{-2x}$ condoning bracketing issues

Method 2: multiplies out and **uses the product rule** on each term of $x^2\mathrm{e}^{-2x}-x^3\mathrm{e}^{-2x}$
Condone issues in the signs of the last two terms for the method mark
Uses the product rule for $uvw=u'vw+uv'w+uvw'$ applied as in method 1

Method 3:Uses **the quotient rule** with $u=x^2-x^3$ and $v=e^{2x}$. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $\frac{vu'-uv'}{v^2}$ If the rule is not quoted nor implied accept expressions of the form $\frac{\mathrm{e}^{2x}\left(Ax-Bx^2\right)-\left(x^2-x^3\right)\times C\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}\right)^2}$

condoning missing brackets on the numerator and e^{2x^2} on the denominator.

Method 4: Apply implicit differentiation to $y\mathrm{e}^{2x}=x^2-x^3 \Rightarrow \mathrm{e}^{2x}\times\frac{\mathrm{d}y}{\mathrm{d}x}+y\times 2\mathrm{e}^{2x}=2x-3x^2$

Condone errors on coefficients and signs

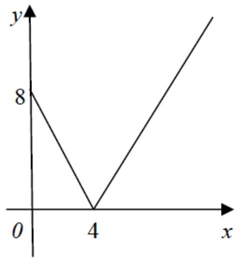
Question Number	Scheme	Marks
6.(a)	$\mathrm{f}(x) > k^2$	B1
		(1)
(b)	$y = \mathrm{e}^{2x} + k^2 \Rightarrow \mathrm{e}^{2x} = y - k^2$	M1
	$\Rightarrow x = \frac{1}{2}\ln(y - k^2)$	dM1
	$\Rightarrow \mathrm{f}^{-1}(x) = \frac{1}{2}\ln(x - k^2), \quad x > k^2$	A1
		(3)
(c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 8x^6 = 6$	M1
	$\Rightarrow 8x^6 = \mathrm{e}^6 \Rightarrow x = ..$	M1
	$\Rightarrow x = \left(\frac{\mathrm{e}}{\sqrt[6]{8}}\right) = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	A1
		(4)
(d)	$\mathrm{fg}(x) = e^{2\times\ln(2x)} + k^2$	M1
	$\Rightarrow \mathrm{fg}(x) = (2x)^2 + k^2 = 4x^2 + k^2$	A1
		(2)
(e)	$\mathrm{fg}(x) = 2k^2 \Rightarrow 4x^2 + k^2 = 2k^2$	
	$\Rightarrow 4x^2 = k^2 \Rightarrow x = ..$	M1
	$\Rightarrow x = \frac{k}{2}$ **only**	A1
		(2) **12 marks**
(alt c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$ $\Rightarrow \ln 2 + \ln x + \ln 2 + 2\ln x + \ln 2 + 3\ln x = 6$ $\Rightarrow 3\ln 2 + 6\ln x = 6$	M1
	$\Rightarrow \ln x = 1 - \frac{1}{2}\ln 2$	M1
	$\Rightarrow x = e^{1-\frac{1}{2}\ln 2}, = \frac{\mathrm{e}}{\sqrt{2}}$ (Ignore any reference to $-\frac{\mathrm{e}}{\sqrt{2}}$)	M1, A1
(alt e)		(4)
	$\Rightarrow 2\ln(2x) = \ln(2k^2 - k^2)$	
	$\Rightarrow \ln(2x)^2 = \ln(k^2), \Rightarrow 4x^2 = k^2 \Rightarrow x = \frac{k}{2}$	M1, A1

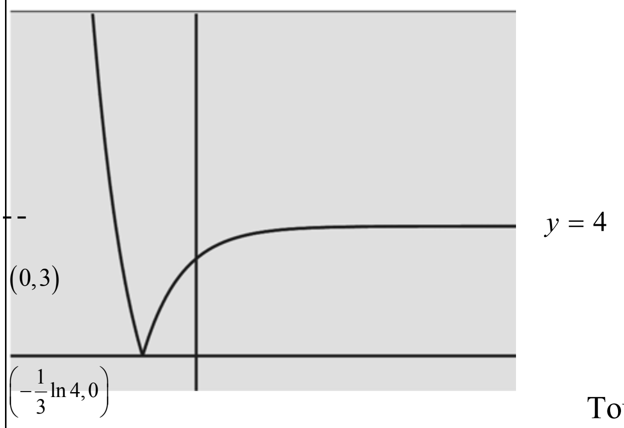
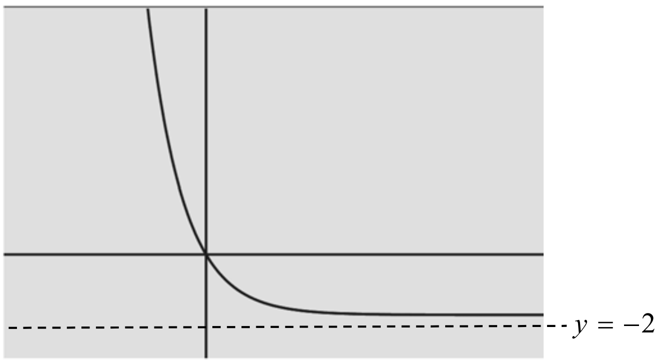
Question Number	Scheme	Marks
5.(a)	$x^2+x-6=(x+3)(x-2)$	B1
	$\dfrac{x}{x+3}+\dfrac{3(2x+1)}{(x+3)(x-2)}=\dfrac{x(x-2)+3(2x+1)}{(x+3)(x-2)}$	M1
	$=\dfrac{x^2+4x+3}{(x+3)(x-2)}$	A1
	$=\dfrac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}(x-2)}$	
	$=\dfrac{(x+1)}{(x-2)}$ cso	A1*
		(4)
(b)	One end either $(y)>1,(y)\geqslant 1$ or $(y)<4,(y)\leqslant 4$	B1
	$1<y<4$	B1
		(2)
(c)	Attempt to set Either $g(x)=x$ or $g(x)=g^{-1}(x)$ or $g^{-1}(x)=x$ or $g^2(x)=x$	
	$\dfrac{(x+1)}{(x-2)}=x$ $\quad \dfrac{x+1}{x-2}=\dfrac{2x+1}{x-1}$ $\quad \dfrac{2x+1}{x-1}=x$ $\quad \dfrac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$	M1
	$x^2-3x-1=0\Rightarrow x=\ldots$	A1, dM1
	$a=\dfrac{3+\sqrt{13}}{2}$ oe $\left(1.5+\sqrt{3.25}\right)$ cso	A1
		(4)
		(10 marks)

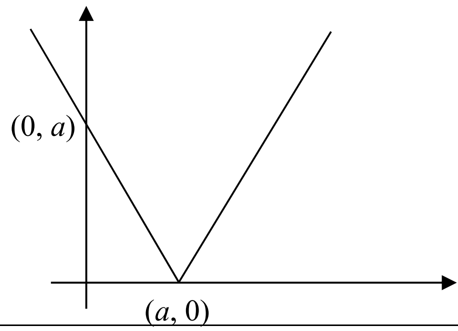
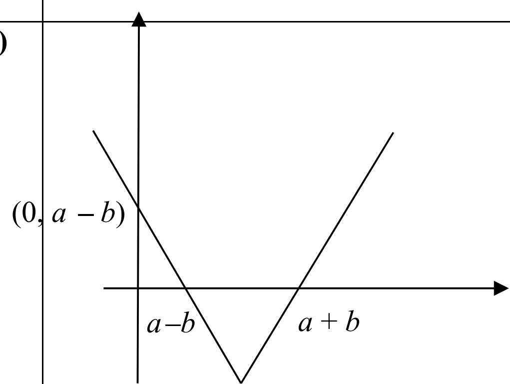
Question Number	Scheme	Marks
7(a)	$0 \leqslant f(x) \leqslant 10$	B1
		(1)
(b)	$ff(0) = f(5), \ = 3$	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y-4 = xy+3x$	M1
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	M1A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5-2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to 7(d)	$gf(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax+b = x-2$ *or* $5-2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = ...$	M1
	$\Rightarrow x = 0.4$ oe	A1 (5)

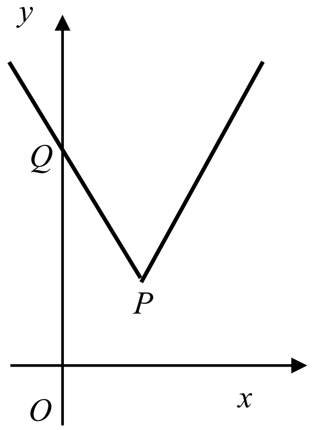
Question	Scheme	Marks	AOs
11(a)	$x = -4$ **or** $y = -5$	B1	1.1b
	$P(-4, -5)$	B1	2.2a
		(2)	
(b)	$3x + 40 = -2(x + 4) - 5 \Rightarrow x = ...$	M1	1.1b
	$x = -10.6$	A1	2.1
		(2)	
(c)	$a > 2$	B1	2.2a
	$y = ax \Rightarrow -5 = -4a \Rightarrow a = \frac{5}{4}$	M1	3.1a
	$\{a : a \leqslant 1.25\} \cup \{a : a > 2\}$	A1	2.5
		(3)	
			(7 marks)

Question Number	Scheme	Marks
4(a)	$\text{fg}(1) = \text{f}(2) = 7$	M1A1
		(2)
(b)	Either g(0)=3 or $g(x \to \infty) \to 0.5$	M1
	$0.5 < \text{g}(x) \leqslant 3$	A1
		(2)
(c)	Attempt change of subject of $y = \dfrac{x+9}{2x+3} \Rightarrow y(2x+3) = x+9$	
	$\Rightarrow 2xy - x = 9 - 3y$	M1
	$\Rightarrow x(2y-1) = 9 - 3y \Rightarrow x = \dfrac{9-3y}{2y-1}$	dM1
	$\text{g}^{-1}(x) = \dfrac{9-3x}{2x-1}, \quad 0.5 < x \leqslant 3$	A1, B1 ft
		(4)
(d)	Attempts f(0) = 2×3+5=11 $\Rightarrow k \leqslant 11$ Or f(3) = 2×0+5=5 $\Rightarrow k > 5$	M1A1
	$5 < k \leqslant 11$	A1
		(3)
		(11 marks)

Question Number	Scheme	Marks
3(a)	 V shape just in Quad 1and correct position	B1
	Meets/cuts y axis at (0,8)	B1
	Meets x axis at (4,0)	B1
		(3)
(b)	$x = 1$	B1
	$x + 5 = -(8 - 2x) \Rightarrow x = 13$	M1A1
		(3)
(c)	$\text{fg}(5) = \text{f}(2) = -1$	M1A1
		(2)
(d)	$\text{f}'(x) = 2x - 3 \Rightarrow \text{min at } x = \frac{3}{2} \Rightarrow \text{min} = -\frac{5}{4}$	M1A1
	Maximum value = 5	B1
	$-\frac{5}{4} \leqslant \text{f}(x) \leqslant 5$	A1
		(4)
		(12 marks)

Question Number	Scheme	Marks
11(a)		
	Shape	B1
	Asymptote $y = 4$	B1
	y intercept $(0, 3)$	B1
	Touches x axis at $\left(-\frac{1}{3}\ln 4, 0\right)$	B1
		[4]
(b)		
	Shape	B1
	Asymptote $y = -2$	B1
	Passes though origin	B1
		[3]
(c)	$f(x) > -4$	B1
		[1]
(d)	$y = e^{-3x} - 4 \quad \Rightarrow e^{-3x} = y + 4$	M1
	$\Rightarrow -3x = \ln(y+4)$ and $x =$	dM1
	$f^{-1}(x) = -\frac{1}{3}\ln(x+4)$ or $\ln\frac{1}{(x+4)^{\frac{1}{3}}}$, $(x > -4)$ cao	A1
		[3]
(e)	$fg(x) = e^{-3\ln\left(\frac{1}{x+2}\right)} - 4$	M1
	$= (x+2)^3 - 4$, $= x^3 + 6x^2 + 12x + 4$	dM1, A1
		[3]
		(14 marks)

Question Number	Scheme	Notes	Marks
6(a)(i)		V shape with vertex on *x*-axis but **not** at the origin.	B1
		Correct V shape with (0, *a*) or just *a* and (*a*, 0) or just *a* marked in the correct places. Left branch must cross or touch the *y*-axis. Allow coordinates the wrong way round if marked in the correct place.	B1
			(2)
(a)(ii)		Their part (i) translated down (by any amount) but clearly not left or right, or the correct shape i.e. a V with the vertex in 4th quadrant.	B1ft
		A *y*-intercept of *a* – *b* on the positive *y*-axis or intercepts of *a* – *b* and *a* + *b* on the positive *x*-axis with *a* + *b* to the right of *a* – *b*	B1
		A fully correct diagram.	B1
			(3)
(b)	$x-a-b=\frac{1}{2}x \Rightarrow x=...$ **or** $-x+a-b=\frac{1}{2}x \Rightarrow x=...$	Solves $x-a-b=\frac{1}{2}x$ **or** solves $-x+a-b=\frac{1}{2}x$ as far as $x=$… Allow $<$ or $>$ for =.	M1
	$x-a-b=\frac{1}{2}x \Rightarrow x=...$ **and** $-x+a-b=\frac{1}{2}x \Rightarrow x=...$	Solves $x-a-b=\frac{1}{2}x$ **and** solves $-x+a-b=\frac{1}{2}x$ as far as $x=$… Allow $<$ or $>$ for =.	M1
	$\frac{2}{3}(a-b)<x<2(a+b)$	ddM1: Chooses inside region. A1: Allow alternatives e.g. $x<2(a+b)$ **and** $x>\frac{2}{3}(a-b)$, $x<2(a+b) \cap x>\frac{2}{3}(a-b)$, $\left(\frac{2}{3}(a-b),\ 2(a+b)\right)$ but not $x<2(a+b)$, $x>\frac{2}{3}(a-b)$	ddM1A1
			(4)
			(9 marks)

Qu	Scheme	Marks
7 (a)(i)	Substitute (0, 5) to give $\lvert b \rvert = 5$ so $b = \pm 5$	B1
	Substitute $\left(\frac{1}{3}, 0\right)$ to give $\lvert \frac{1}{3}a + b \rvert = 0$ so $a = \mp 15$	M1 A1
(ii)	Gives equation as $y = \lvert -15x + 5 \rvert$ or $y = \lvert 15x - 5 \rvert$	B1
		(4)
(b)	 V shape correct way up P at $\left(\frac{2}{3}, 3\right)$ Q at (0, 8)	B1 B1 B1
		(3)
		(7 marks)

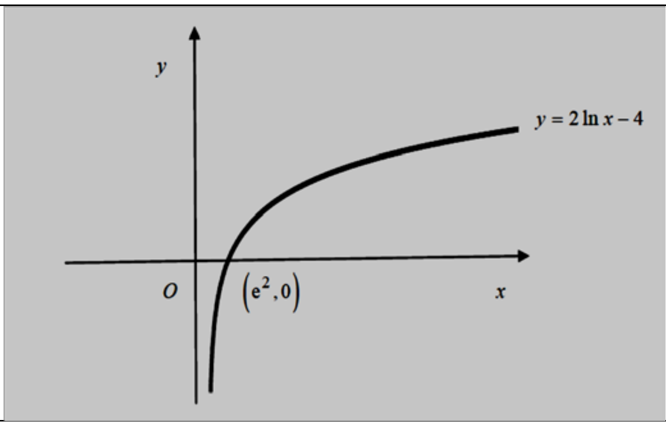
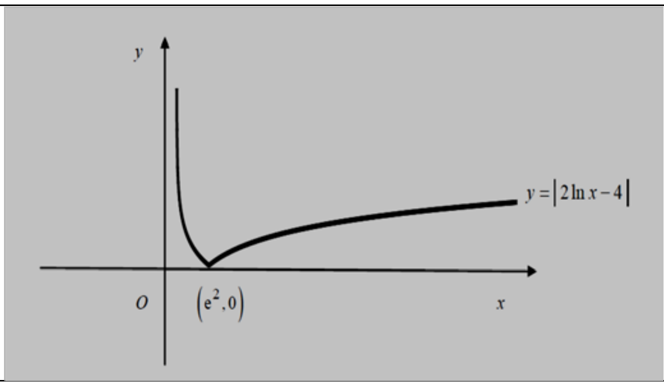
(a)
(i)
B1: For both $b = \pm 5$ not just $\lvert b \rvert = 5$
M1: Substitute $\left(\frac{1}{3}, 0\right)$ to give $\lvert \frac{1}{3}a + b \rvert = 0$. This mark is implied by $a = \pm 3 \times$ value of b
A1: $a = 15$ corresponding to $b = -5$ **and** $a = -15$ corresponding to $b = 5$

If they write down an equation rather than giving values of a and b then
Just $y = \lvert -15x + 5 \rvert$ **or** $y = \lvert 15x - 5 \rvert$ scores B0, M1, A0
Both $y = \lvert -15x + 5 \rvert$ **and** $y = \lvert 15x - 5 \rvert$ scores B1, M1, A1
Linear equations $y = -15x + 5$ and/or $y = 15x - 5$ (without the modulus) only score B0 M1 A0

(ii) Note that this is an A1 mark on e-pen

B1: $y = \lvert -15x + 5 \rvert$ or $y = \lvert 15x - 5 \rvert$ or allow equations such as for this mark only $\mathrm{f}(x) = \begin{cases} 15x - 5 & x \geqslant \frac{1}{3} \\ -15x + 5 & x < \frac{1}{3} \end{cases}$

If candidates don't state (i), (ii) and write down just $y = \lvert -15x + 5 \rvert$ they would score (i) B0 M1 A0 (ii) B1
(b) There must be a sketch to score any of these marks.
B1: V shape the correct way up any position but not on the x- axis. Accept V's that don't have symmetry
B1: P at $\left(\frac{2}{3}, 3\right)$ Score if the coordinates are stated within the text OR marked on the axes. If they appear in both then the graph takes precedence.
B1: For **crossing** the y- axis at (0, 8). Accept 8 marked on the correct axis. Condone (8,0) marked on the correct axis

Question Number	Scheme		Marks
9(a)(i)		Shape	B1
		$\left(e^2,0\right)$	B1
		Asymptote $x=0$	B1
			(3)
(a)(ii)		Shape	B1ft
		Asymptote and coordinate	B1ft
			(2)
(b)	$2\ln x-4=4\Rightarrow \ln x=4\Rightarrow x=e^4$		M1A1
	$2\ln x-4=-4\Rightarrow \ln x=0\Rightarrow x=1$		M1A1
			(4)
(c)	$gf(x)=e^{2\ln x-4+5}-2,=e^1\times e^{2\ln x}-2=ex^2-2$		M1,dM1A1
			(3)
(d)	$gf(x)>-2$		B1
			(1)
			(13 marks)

(a)(i)

B1: For a logarithmic shaped curve in any position. For this mark be tolerant on slips of the pen at either end. See Practice and Qualification for examples.

B1: Intersection with the x axis at $\left(e^2,0\right)$.

Allow e^2 marked on the x axis. Condone $\left(0,e^2\right)$ being marked on the positive x axis.

Do not allow e^2 appearing as 7.39 for this mark unless e^2 is seen in the body of the script.
Allow if the coordinate is given in body of script. If they are given in the body of the script and differently on the curve (save for the decimal equivalent) then the ones on the curve take precedence.

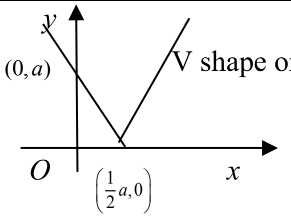
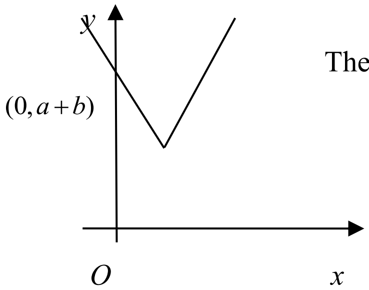
B1: **Equation** of asymptote is $x=0$ (do not allow “ y-axis”). Note that the curve must appear to have an asymptote at $x=0$

(a)(ii)

B1ft: For either the correct shape or a reflection of their “negative” curve in (a) in the x-axis. For this to be scored it must have appeared both above and below the x-axis. The curve to the lhs of the intercept must appear to have the correct curvature

B1ft: Score for the correct coordinates and asymptote. Alternatively follow through on the coordinates and asymptote given in part (a) as long as the curve appeared both above and below the x-axis and the curve approaches the same asymptote stated in (a)(i). Do not penalise “y-axis” given as the asymptote twice – i.e. penalise in (a)(i) only.

If the curves are sketched on the same axes – it must be clear which curve is which – if in doubt use review.

Question Number	Scheme	Marks
6.(a)(i)	 V shape on x - axis **or** coordinates $\left(\frac{1}{2}a,0\right)$ **and** $(0,a)$	B1
	Correct shape, position and coordinates	B1
(ii)	 Their "V" shape translated up or $(0,a+b)$	B1ft
	Correct shape, position and $(0,a+b)$	B1
		(4)
(b)	States or uses $a+b=8$	B1
	Attempts to solve $\lvert 2x-a\rvert+b=\frac{3}{2}x+8$ in either x or with $x=c$	
	$2c-a+b=\frac{3}{2}c+8 \Rightarrow kc=\mathrm{f}(a,b)$	M1
	Combines $kc=\mathrm{f}(a,b)$ with $a+b=8 \quad \Rightarrow c=4a$	dM1 A1
		(4)
		(8 marks)

(a)(i)

B1 V shape sitting anywhere on the x- axis **or** for $\left(\frac{1}{2}a,0\right)$ and $(0,a)$ lying on the curve.

Condone non -symmetrical graphs and ones lying on just one side of the y -axis

B1 V shape sitting on the positive x-axis at $\left(\frac{1}{2}a,0\right)$, cutting the y-axis at $(0,a)$ and lying in both quadrants 1 and 2

Accept $\frac{1}{2}a$ and a marked on the correct axis. Condone say $(a,0)$ for $(0,a)$ as long as it is on the correct axis.

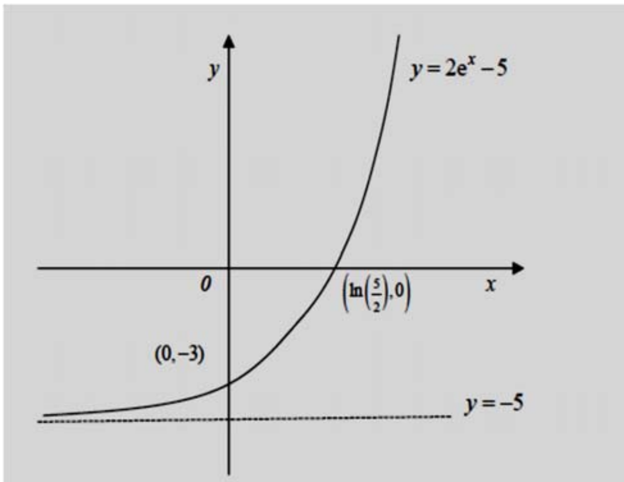
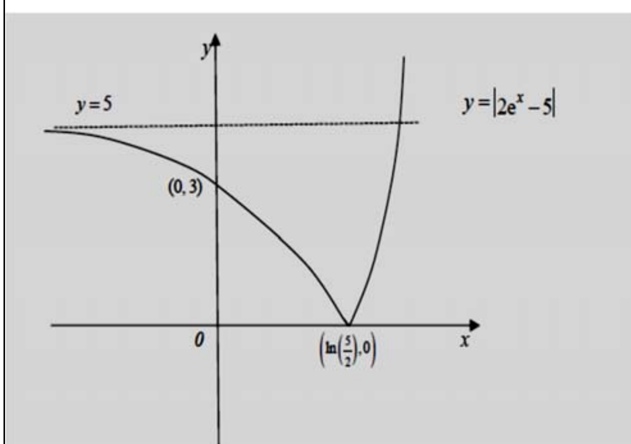
Condone a dotted line appearing on the diagram as many reflect $y=2x-a$ to sketch $y=\lvert 2x-a\rvert$

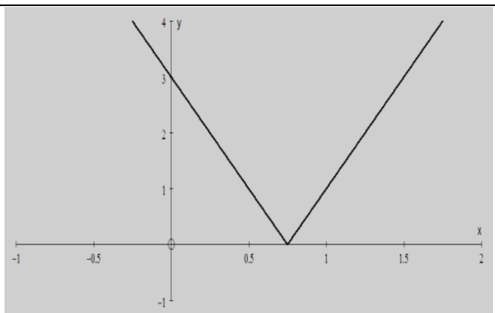
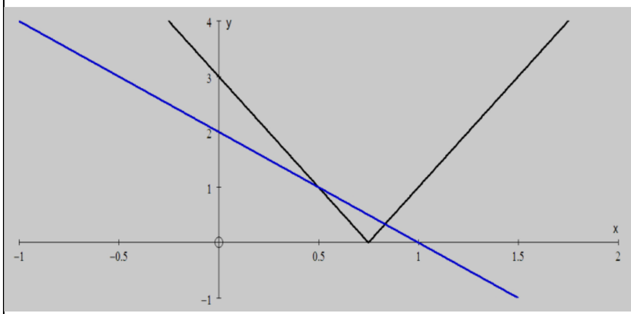
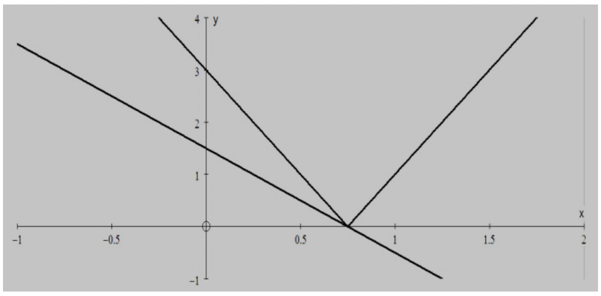
If it is a solid line then it would not score the shape mark.

(a)(ii)

B1ft Follow through on (a)(i). Their graph translated up. Allow on U shapes and non symmetrical graphs.
Alternatively score for the $(0,a+b)$ lying on the curve

B1 V shape lying in quadrants 1 and 2 with the vertex in quadrant 1 cutting the y- axis at $(0,a+b)$
Ignore any coordinates given for the vertex.

Question Number	Scheme	Marks
2.(ai)		
	Shape	B1
	$\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$	B1
	$y=-5$	B1
		(3)
(aii)		
	Shape inc cusp	B1ft
	$\left(\ln\left(\frac{5}{2}\right),0\right)$ *and* $(0,3)$	B1ft
	$y=5$	B1ft
		(3)
(b)	$x \geqslant \ln\left(\frac{5}{2}\right)$	B1 ft
		(1)
(c)	$2e^x-5=-2 \Rightarrow (x)=\ln\left(\frac{3}{2}\right)$	M1A1
	$(x)=\ln\left(\frac{7}{2}\right)$	B1
		(3)
		(10 marks)

Question Number	Scheme	Marks
5. (a)	 V shaped graph	B1
	Touches x axis at ¾ and cuts y axis at 3	B1
		(2)
(b)	 Solves $4x-3=2-2x$ or $3-4x=2-2x$ to give either value of x	M1
	Both $x=\frac{5}{6}$ and $x=\frac{1}{2}$ or $x>\frac{5}{6}$ or $x<\frac{1}{2}$	A1
	$x<\frac{1}{2}$ or $x>\frac{5}{6}$	dM1A1
		(4)
(c)	 Draws graph Or solves $\lvert 4x-3\rvert=1\frac{1}{2}-2x$ to give one soln x = ¾	M1
	Accept for all values of x except $x=\frac{3}{4}$ Or $(x\in\mathbb{R},)$ $x\neq\frac{3}{4}$, or $x<\frac{3}{4}$, $x>\frac{3}{4}$	A1
		(2)
		(8 marks)

(a)

B1 A 'V' shaped graph. The position is not important. Do not accept curves. See practice and qualification items for clarity. Accept a V shape with a 'dotted' extension of $y = 4x - 3$ appearing under the x axis.

B1 The graph **meets** the x axis at $x = \frac{3}{4}$ and **crosses** the y axis at $y = 3$. Do not allow multiple meets or crosses

If they have lost the previous B1 mark for an extra section of graph underneath the x axis allow for **crossing** the x axis at $x = \frac{3}{4}$ and **crosses** the y axis at $y = 3$.

Accept marked elsewhere on the page with A and B marked on the graph and $A = \left(\frac{3}{4}, 0\right)$ and $B = (0,3)$

Condone $\left(0, \frac{3}{4}\right)$ and (3, 0) marked on the correct axis

(b)

M1 Attempts to solve $|4x-3| ... 2-2x$ finding at least one solution. You may see … replaced by either = or >

Accept as evidence $\pm 4x \pm 3 = 2 - 2x \Rightarrow x = ..$

Accept as evidence $\pm 4x \pm 3 > 2 - 2x \Rightarrow x > ..,$ or $x < ..$

A1 Both critical values $x = \frac{5}{6}$ and $x = \frac{1}{2}$, or one inequality, accept $x > \frac{5}{6}$ or $x < \frac{1}{2}$

Accept $x = 0.83$ and $x = 0.5$ for the critical values

Accept both of these answers with no incorrect working for both marks

dM1 Dependent upon the previous M, this is scored for selecting the outside region of their two points.

Eg if M1 has been scored for $4x - 3 = 2 - 2x \Rightarrow x = 0.83$ and $-4x - 3 = 2 - 2x \Rightarrow x = -2.5$

A correct application of M1 would be $x < -2.5, x > 0.83$

A1 Correct answer only $x < \frac{1}{2}$ or $x > \frac{5}{6}$.

Accept $x < 0.5, x > 0.8\dot{3}$

(c)

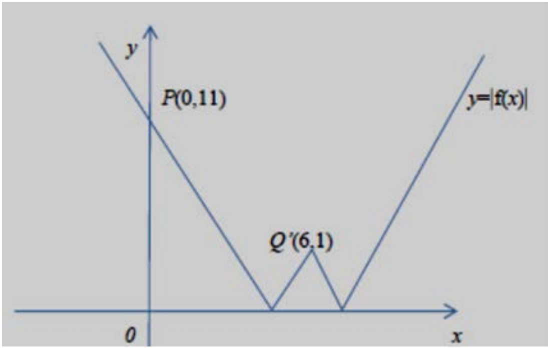
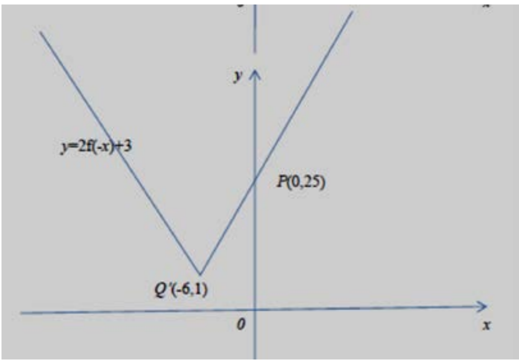
M1 **Either** sketch both lines showing a single intersection at the point $x = \frac{3}{4}$

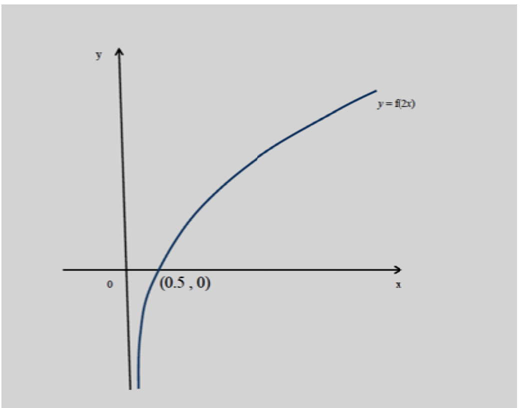
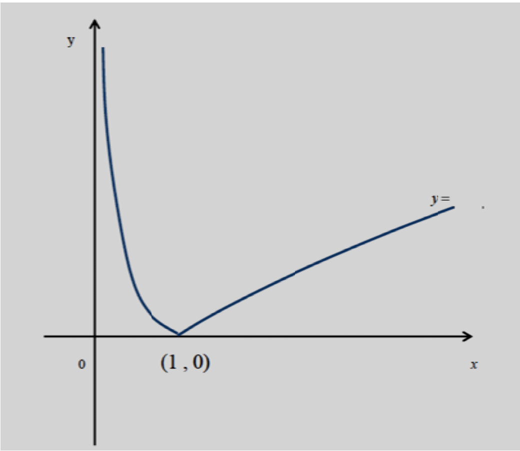
Or solves $|4x-3| = 1\frac{1}{2} - 2x$ using both $4x - 3 = 1\frac{1}{2} - 2x$ and $-4x + 3 = 1\frac{1}{2} - 2x$ **giving one solution** $x = \frac{3}{4}$

Accept $|4x-3| > 1\frac{1}{2} - 2x$ using both $4x - 3 > 1\frac{1}{2} - 2x$ and $-4x + 3 > 1\frac{1}{2} - 2x$ **giving one solution** $x ... \frac{3}{4}$

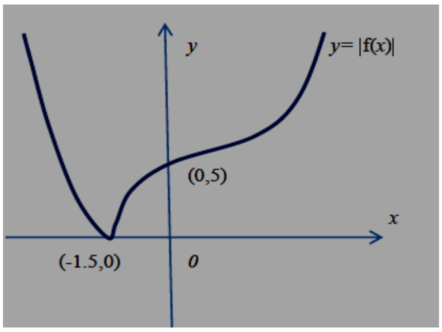
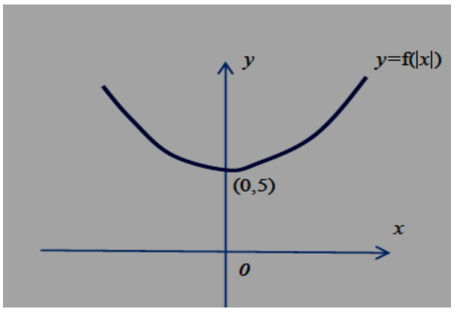
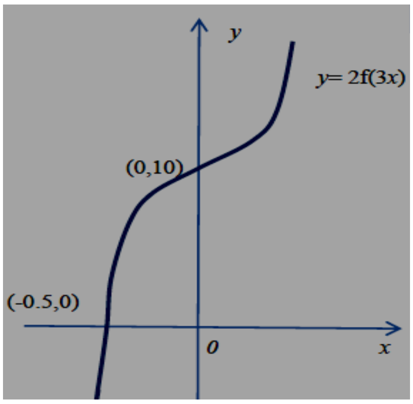
If two values are obtained using either method it is M0A0

A1 States that the solution set is all values apart from $x = \frac{3}{4}$. Do not isw in this question. Score their final statement. Accept versions of all values of x except $x = \frac{3}{4}$ or $x \in \mathbb{R}$, $x \neq \frac{3}{4}$, or $x < \frac{3}{4}$, $x > \frac{3}{4}$

Question Number	Scheme	Marks
4.(a)	 'W' Shape (0, 11) and (6, 1)	B1 B1 (2)
(b)	 'V' shape (-6,1) (0,25)	B1 B1 B1 (3)
(c)	One of $a = 2$ or $b = 6$ $a = 2$ and $b = 6$	B1 B1 (2)
		(7 marks)

Question Number	Scheme		Marks
2.(a)		Shape (0.5, 0)	B1 B1 (2)
(b)		Shape (1,0) Cusp at (1,0)	B1 B1 B1 (3)
			(5 marks)

Question Number	Scheme	Marks
4.(a)	$f(x) \geqslant 3$	M1A1
		(2)
(b)	An attempt to find $2\|3-4x\|+3$ when $x=1$	M1
	Correct answer $fg(1)=5$	A1
		(2)
(c)	$y=3-4x \Rightarrow 4x=3-y \Rightarrow x=\frac{3-y}{4}$	M1
	$g^{-1}(x)=\frac{3-x}{4}$	A1
		(2)
(d)	$[g(x)]^2=(3-4x)^2$	B1
	$gg(x)=3-4(3-4x)$	M1
	$gg(x)+[g(x)]^2=0 \Rightarrow -9+16x+9-24x+16x^2=0$	
	$16x^2-8x=0$	A1
	$8x(2x-1)=0 \Rightarrow x=0, 0.5$ oe	M1A1
		(5)
		(11 marks)

Question Number	Scheme		Marks
4.(a)		Shape including cusp (-1.5, 0) **and** (0, 5)	B1 B1 (2)
(b)		Shape (0,5)	B1 B1 (2)
(c)		Shape (0,10) (-0.5, 0)	B1 B1 B1 (3) **(7 marks)**

(a) **Note that this appears as M1A1 on EPEN**

B1 Shape (inc cusp) with graph in just quadrants 1 and 2. Do not be overly concerned about relative gradients, but the left hand section of the curve should not bend back beyond the cusp

B1 This is independent, and for the curve touching the *x*-axis at (-1.5, 0) **and** crossing the *y*-axis at (0,5)

(b) **Note that this appears as M1A1 on EPEN**

B1 For a U shaped curve symmetrical about the *y*- axis

B1 (0,5) lies on the curve

(c) **Note that this appears as M1B1B1 on EPEN**

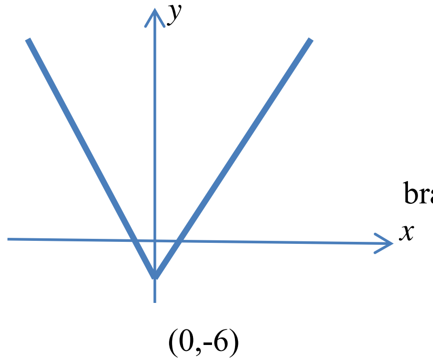
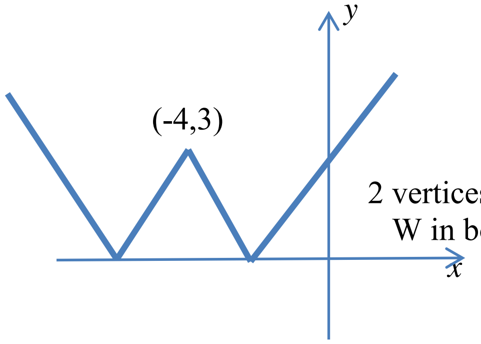
B1 Correct shape- do not be overly concerned about relative gradients. Look for a similar shape to f(*x*)

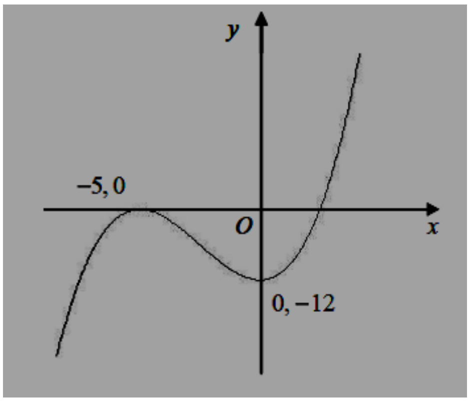
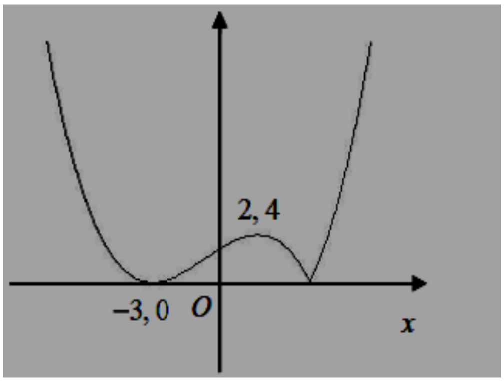
B1 Curve **crosses** the *y* axis at (0, 10). The curve must appear in both quadrants 1 and 2

B1 Curve **crosses** the *x* axis at (-0.5, 0). The curve must appear in quadrants 3 and 2.

In all parts accept the following for any co-ordinate. Using (0,3) as an example, accept both (3,0) or 3 written on the *y* axis (as long as the curve passes through the point)

Special case with (a) and (b) completely correct but the wrong way around mark - SC(a) 0,1 SC(b) 0,1
Otherwise follow scheme

Question Number	Scheme	Marks
3 (a)		
	V shape	B1
	vertex on y axis &both branches of graph cross x axis	B1
	'y' co-ordinate of R is -6	B1
		(3)
(b)		
	W shape	B1
	2 vertices on the negative x axis. W in both quad 1 & quad 2.	B1dep
	R'=(-4,3)	B1
		(3)
		6 Marks
4 (a)	$y = 4 - \ln(x+2)$ $\ln(x+2) = 4 - y$ $x + 2 = e^{4-y}$ $x = e^{4-y} - 2$	M1
	$f^{-1}(x) = e^{4-x} - 2$ oe	M1A1
		(3)
(b)	$x \leq 4$	B1
		(1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$	M1
	$fg(x) = 4 - x^2$	dM1A1
		(3)
(d)	$fg(x) \leq 4$	B1ft
		(1)
		8 Marks

Question No	Scheme		Marks
2	(a)		
		Shape x coordinates correct y coordinates correct	B1 B1 B1 (3)
	(b)		
		Shape Max at (2,4) Min at (-3,0)	B1 B1 B1 (3)
			6 marks

(a)

B1 Shape unchanged. The positioning of the curve is not significant for this mark. The right hand section of the curve does not have to cross x axis.

B1 The x- coordinates of P' and Q' are -5 and 0 respectively. This is for translating the curve 2 units left. The minimum point Q' must be on the y axis. Accept if -5 is marked on the x axis for P' with Q' on the y axis (marked -12).

B1 The y- coordinates of P' and Q' are 0 and -12 respectively. This is for the stretch ×3 parallel to the y axis. The maximum P' must be on the x axis. Accept if -12 is marked on the y axis for Q' with P' on the x axis (marked -5)

(b)

B1 The curve below the x axis reflected in the x axis and the curve above the x axis is unchanged. Do not accept if the curve is clearly rounded off with a zero gradient at the x axis but allow small curvature issues. Use the same principles on the lhs- do not accept if this is a cusp.

B1 Both the x- and y- coordinates of Q', (2,4) given correctly and associated with the maximum point in the first quadrant. To gain this mark there must be a graph and it must only have one maximum.
Accept as 2 and 4 marked on the correct axes or in the script as long as there is no ambiguity.

B1 Both the x- and y- coordinates of P', (-3,0) given correctly and associated with the minimum point in the second quadrant. To gain this mark there must be a graph. Tolerate two cusps if this mark has been lost earlier in the question. Accept (0, -3) marked on the correct axis.

Question No	Scheme	Marks
3	(a) 20 (mm^2)	**B1** **(1)** **M1**

	(b) $'40' = 20\ e^{1.5t} \quad \rightarrow \quad e^{1.5t} = c$ $e^{1.5t} = \frac{40}{20} = (2)$ Correct order $1.5t = ln'2' \rightarrow \quad t = \frac{lnc}{1.5}$ $t = \frac{ln2}{1.5} = (\ awrt\ 0.46)$ 12.28 or 28 (minutes}	**A1** **M1** **A1** **A1** **(5)** **(6 marks)**

(a)

B1 Sight of 20 relating to the value of A at t=0. Do not worry about (incorrect) units. Accept its sight in (b)

(b)

M1 Substitutes A=40 or twice their answer to (a) **and** proceeds to $e^{1.5t} = constant$. Accept non numerical answers.

A1 $e^{1.5t} = \frac{40}{20}\ or\ 2$

M1 Correct ln work to find t. Eg $e^{1.5t} = constant \rightarrow 1.5t = \ln(constant) \rightarrow t =$
The order must be correct. Accept non numerical answers**. See below for correct alternatives**

A1 Achieves either $\frac{\ln(2)}{1.5}$ or awrt 0.46 2sf

A1 Either 12:28 or 28 (minutes). Cao

Alternatives in (b)

Alt 1- taking ln's of both sides on line 1

M1 Substitutes A=40, or twice (a) takes ln's of both sides **and** proceeds to $\ln('40') = ln20 + lne^{1.5t}$

A1 $\ln(40) = ln20 + 1.5t$

M1 Make t the subject with correct *ln* work.

$$\ln('40') - ln20 = 1.5t\ \ or\ \ \ln\left(\frac{'40'}{20}\right) = 1.5t \rightarrow\ t =$$

A1,A1 are the same

Alt 2- trial and improvement-hopefully seen rarely

M1 Substitutes t= 0.46 and t=0.47 into $20e^{1.5t}$ to obtain A at both values. Must be to at least 2dp but you may accept tighter interval but the interval must span the correct value of 0.46209812

A1 Obtains A(0.46)=39.87 AND A(0.47)=40.47 or equivalent

M1 Substitutes t=0.462 and t=0.4625 into $40e^{1.5t}$

A1 Obtains A(0.462)=39.99 AND A(0.4625)=40.02 or equivalent and states t=0.462 (3sf)

A1 AS ABOVE

No working leading to fully correct accurate answer (3sf or better) send/escalate to team leader