4. (i) Show that $\sum_{r=1}^{16}\left(3 + 5r + 2^r\right) = 131\,798$

(4)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3)

13. A sequence of numbers $a_1, a_2, a_3, \dots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

15. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

Leave blank

5. A sequence is defined by

$$u_1 = 3$$

$$u_{n+1} = 2 - \frac{4}{u_n}, \quad n \geqslant 1$$

Find the exact values of

(a) u_2, u_3 and u_4

(3)

(b) u_{61}

(1)

(c) $\sum_{i=1}^{99} u_i$

(3)

Leave blank

11. The first three terms of an arithmetic series are 60, $4p$ and $2p - 6$ respectively.

(a) Show that $p = 9$

(2)

(b) Find the value of the 20th term of this series.

(3)

(c) Prove that the sum of the first n terms of this series is given by the expression

$$12n\,(6 - n)$$

(3)

Leave blank

9. (i) Find the value of $\sum_{r=1}^{20} (3 + 5r)$

(3)

(ii) Given that $\sum_{r=0}^{\infty} \frac{a}{4^r} = 16$, find the value of the constant a.

(4)

Leave blank

5. (a) Prove that the sum of the first n terms of an arithmetic series is given by the formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a is the first term of the series and d is the common difference between the terms.

(4)

(b) Find the sum of the integers which are divisible by 7 and lie between 1 and 500

(3)

Leave blank

8. A sequence is defined by

$$u_1 = k$$
$$u_{n+1} = 3u_n - 12, \qquad n \geqslant 1$$

where k is a constant.

(a) Write down fully simplified expressions for u_2, u_3 and u_4 in terms of k.
(4)

Given that $u_4 = 15$

(b) find the value of k,
(2)

(c) find $\sum_{i=1}^{4} u_i$, giving an exact numerical answer.
(3)

Leave blank

10. A sequence is defined by

$$u_1 = 4$$
$$u_{n+1} = \frac{2u_n}{3}, \qquad n \geqslant 1$$

(a) Find the exact values of u_2, u_3 and u_4

(2)

(b) Find the value of u_{20}, giving your answer to 3 significant figures.

(2)

(c) Evaluate

$$12 - \sum_{i=1}^{16} u_i$$

giving your answer to 3 significant figures.

(3)

(d) Explain why $\sum_{i=1}^{N} u_i < 12$ for all positive integer values of N.

(1)

Leave blank

5. (i) $$U_{n+1} = \frac{U_n}{U_n - 3}, \quad n \geqslant 1$$

Given $U_1 = 4$, find

(a) U_2

(1)

(b) $\sum_{n=1}^{100} U_n$

(2)

(ii) Given

$$\sum_{r=1}^{n} (100 - 3r) < 0$$

find the least value of the positive integer n.

(3)

Leave blank

6. A sequence is defined by

$$u_1 = 36$$
$$u_{n+1} = \frac{2}{3}u_n, \qquad n \geqslant 1$$

(a) Find the exact simplified values of u_2, u_3 and u_4 **(2)**

(b) Write down the common ratio of the sequence. **(1)**

(c) Find, giving your answer to 4 significant figures, the value of u_{11} **(2)**

(d) Find the exact value of $\sum_{i=1}^{6} u_i$ **(2)**

(e) Find the value of $\sum_{i=1}^{\infty} u_i$ **(2)**

Leave blank

4. An arithmetic series has first term a and common difference d.

Given that the sum of the first 9 terms is 54

(a) show that

$$a + 4d = 6$$

(2)

Given also that the 8th term is half the 7th term,

(b) find the values of a and d.

(4)

Leave blank

9. The first three terms of a geometric sequence are

$$7k - 5,\ \ 5k - 7,\ \ 2k + 10$$

where k is a constant.

(a) Show that $11k^2 - 130k + 99 = 0$

(4)

Given that k is not an integer,

(b) show that $k = \dfrac{9}{11}$

(2)

For this value of k,

(c) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

(6)

Leave blank

5. (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162

Find

(a) the common ratio,

(4)

(b) the first term.

(2)

(ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290

(4)

Leave blank

6. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$

The sum to infinity of the series is S_∞

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which

$$S_\infty - S_N < 0.5$$

(4)

Leave blank

5. The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where p is a **positive** constant.

(a) Show that $11p^2 - 10p - 225 = 0$

(4)

(b) Hence show that $p = 5$

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer.

(3)

12. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.

(4)

5. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

Leave blank

9. In the first month after opening, a mobile phone shop sold 300 phones. A model for future sales assumes that the number of phones sold will increase by 5% per month, so that 300×1.05 will be sold in the second month, 300×1.05^2 in the third month, and so on.

Using this model, calculate

(a) the number of phones sold in the 24th month,

(2)

(b) the total number of phones sold over the whole 24 months.

(2)

This model predicts that, in the Nth month, the number of phones sold in that month exceeds 3000 for the first time.

(c) Find the value of N.

(3)

Leave blank

12. A business is expected to have a yearly profit of £275 000 for the year 2016. The profit is expected to increase by 10% per year, so that the expected yearly profits form a geometric sequence with common ratio 1.1

(a) Show that the difference between the expected profit for the year 2020 and the expected profit for the year 2021 is £40 300 to the nearest hundred pounds.

(3)

(b) Find the first year for which the expected yearly profit is more than one million pounds.

(4)

(c) Find the total expected profits for the years 2016 to 2026 inclusive, giving your answer to the nearest hundred pounds.

(3)

Leave blank

8. A 25-year programme for building new houses began in Core Town in the year 1986 and finished in the year 2010.

The number of houses built each year form an arithmetic sequence. Given that 238 houses were built in the year 2000 and 108 were built in the year 2010, find

(a) the number of houses built in 1986, the first year of the building programme,

(5)

(b) the total number of houses built in the 25 years of the programme.

(2)

Leave blank

9. The resident population of a city is 130 000 at the end of Year 1

A model predicts that the resident population of the city will increase by 2% each year, with the populations at the end of each year forming a geometric sequence.

(a) Show that the predicted resident population at the end of Year 2 is 132 600

(1)

(b) Write down the value of the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year which will end with the resident population of the city exceeding 260 000

(c) Show that

$$N > \frac{\log_{10} 2}{\log_{10} 1.02} + 1$$

(4)

(d) Find the value of N.

(1)

Leave blank

14. A geometric series has a first term a and a common ratio r.

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(4)

A liquid is to be stored in a barrel.

Due to evaporation, the volume of the liquid in a barrel at the end of a year is 7% less than the volume at the start of the year.

At the start of the first year, a barrel is filled with 180 litres of the liquid.

(b) Show that the amount of the liquid in this barrel at the end of 5 years is approximately 125.2 litres.

(2)

At the start of each year a new identical barrel is filled with 180 litres of the liquid so that, at the end of 20 years, there are 20 barrels containing varying amounts of the liquid.

(c) Calculate the total amount of the liquid, to the nearest litre, in the 20 barrels at the end of 20 years.

(3)

Leave blank

11. Wheat is to be grown on a farm.

A model predicts that the mass of wheat harvested on the farm will increase by 1.5% per year, so that the mass of wheat harvested each year forms a geometric sequence.

Given that the mass of wheat harvested during year one is 6000 tonnes,

(a) show that, according to the model, the mass of wheat harvested on the farm during year 4 will be approximately 6274 tonnes.
(2)

During year N, according to the model, there is predicted to be more than 8000 tonnes of wheat harvested on the farm.

(b) Find the smallest possible value of N.
(5)

It costs £5 per tonne to harvest the wheat.

(c) Assuming the model, find the total amount that it would cost to harvest the wheat from year one to year 10 inclusive. Give your answer to the nearest £1000.
(3)

Leave blank

6. Each year Lin pays into a savings scheme. In year 1 she pays in £600.
Her payments then increase by £80 a year, so that she pays £680 into the savings scheme in year 2, £760 in year 3 and so on. In year N, Lin pays £1000 into the savings scheme.

(a) Find the value of N.

(2)

(b) Find the total amount that Lin pays into the savings scheme from year 1 to year 15 inclusive.

(2)

Saima starts paying into a different savings scheme at the same time as Lin starts paying into her savings scheme.

In year 1 she pays in £A. Her payments increase by £A each year so that she pays £$2A$ in year 2, £$3A$ in year 3 and so on.

Given that Saima and Lin have each paid, in total, the same amount of money into their savings schemes after 15 years,

(c) find the value of A.

(3)

Leave blank

14. A new mineral has been discovered and is going to be mined over a number of years.

A model predicts that the mass of the mineral mined each year will decrease by 15% per year, so that the mass of the mineral mined each year forms a geometric sequence.

Given that the mass of the mineral mined during year 1 is 8000 tonnes,

(a) show that, according to the model, the mass of the mineral mined during year 6 will be approximately 3550 tonnes.

(2)

According to the model, there is a limit to the total mass of the mineral that can be mined.

(b) With reference to the geometric series, state why this limit exists.

(1)

(c) Calculate the value of this limit.

(2)

It is decided that a total mass of 40 000 tonnes of the mineral is required. This is going to be mined from year 1 to year N inclusive.

(d) Assuming the model, find the value of N.

(5)

Leave blank

3. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

(a) Show that the predicted profit in the year 2016 is £138 915

(1)

(b) Find the first year in which the yearly predicted profit exceeds £200 000

(5)

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

(3)