2. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

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5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

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11.

$$\frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \equiv A + \frac{B}{(x - 3)} + \frac{C}{(1 - 2x)}$$

(a) Find the values of the constants A, B and C.

(4)

$$f(x) = \frac{1 + 11x - 6x^2}{(x - 3)(1 - 2x)} \qquad x > 3$$

(b) Prove that $f(x)$ is a decreasing function.

(3)

3. $$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$

(1)

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12. $$f(x) = 10e^{-0.25x}\sin x, \qquad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

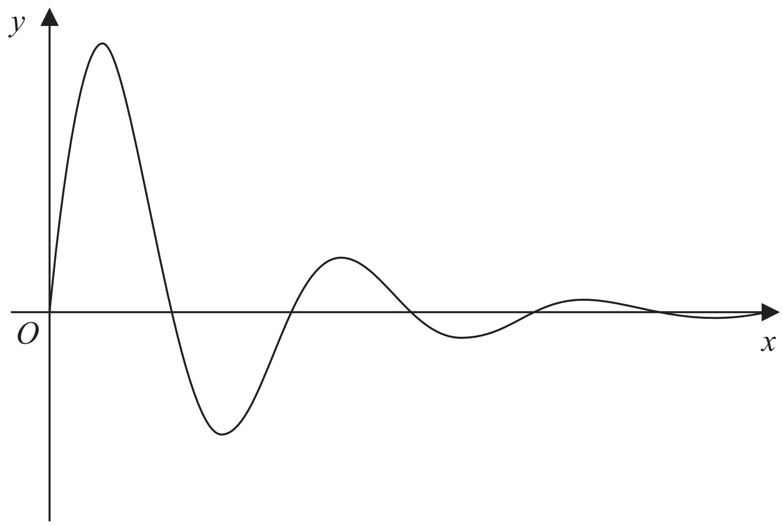

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$$H(t) = \left|10e^{-0.25t}\sin t\right| \qquad t \geqslant 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

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9.

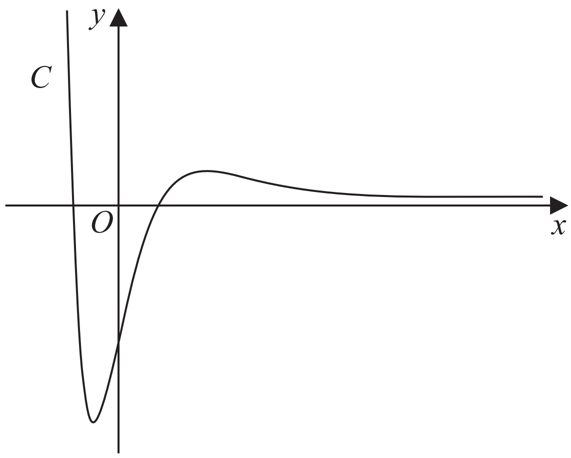

Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8(2 + x - x^2)\mathrm{e}^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

(3)

13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where k is a constant.

(a) Deduce the value of k.

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g.

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

Leave blank

1. $$f(x) = \frac{2x}{x^2 + 3}, \qquad x \in \mathbb{R}$$

Find the set of values of x for which $f'(x) > 0$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Leave blank

3. Given that

$$y = \frac{\cos 2\theta}{1 + \sin 2\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{a}{1 + \sin 2\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where a is a constant to be determined.

(4)

Leave blank

8. The volume V of a spherical balloon is increasing at a constant rate of 250 $\text{cm}^3\,\text{s}^{-1}$.
Find the rate of increase of the radius of the balloon, in $\text{cm}\,\text{s}^{-1}$, at the instant when the volume of the balloon is 12 000 cm^3.
Give your answer to 2 significant figures.

(5)

[*You may assume that the volume V of a sphere of radius r is given by the formula* $V = \dfrac{4}{3}\pi r^3$.]

Leave blank

10. (a) Use the identity for $\sin(A + B)$ to prove that

$$\sin 2A \equiv 2\sin A \cos A$$

(2)

(b) Show that

$$\frac{d}{dx}\left[\ln\left(\tan\left(\tfrac{1}{2}x\right)\right)\right] = \operatorname{cosec} x$$

(4)

A curve C has the equation

$$y = \ln\left(\tan\left(\tfrac{1}{2}x\right)\right) - 3\sin x, \qquad 0 < x < \pi$$

(c) Find the x coordinates of the points on C where $\frac{dy}{dx} = 0$

Give your answers to 3 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

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11.

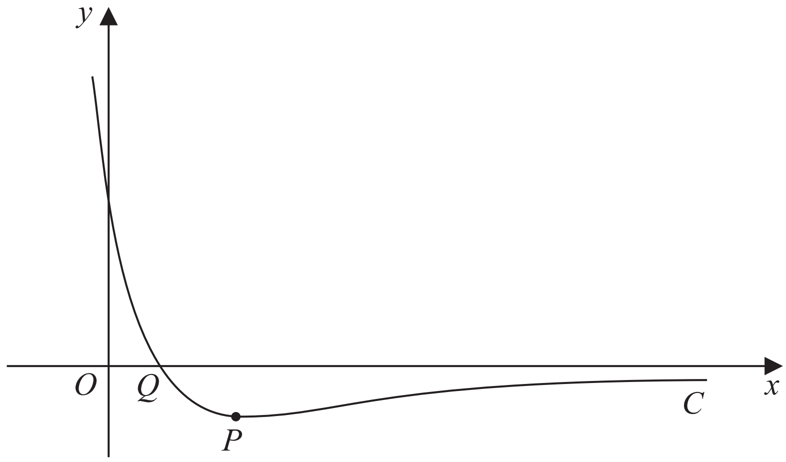

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = e^{a-3x} - 3e^{-x}, \quad x \in \mathbb{R}$$

where a is a constant and $a > \ln 4$

The curve C has a turning point P and crosses the x-axis at the point Q as shown in Figure 2.

(a) Find, in terms of a, the coordinates of the point P.
(6)

(b) Find, in terms of a, the x coordinate of the point Q.
(3)

(c) Sketch the curve with equation

$$y = \left|e^{a-3x} - 3e^{-x}\right|, \quad x \in \mathbb{R}, \ a > \ln 4$$

Show on your sketch the exact coordinates, in terms of a, of the points at which the curve meets or cuts the coordinate axes.
(3)

Leave blank

1. The curve C has equation

$$y = \frac{3x - 2}{(x - 2)^2}, \quad x \neq 2$$

The point P on C has x coordinate 3

Find an equation of the normal to C at the point P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Leave blank

6. (i) Given $x = \tan^2 4y$, $0 < y < \frac{\pi}{8}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x.

Write your answer in the form $\frac{1}{A(x^p + x^q)}$, where A, p and q are constants to be found.

(5)

(ii) The volume V of a cube is increasing at a constant rate of 2 $\mathrm{cm}^3\ \mathrm{s}^{-1}$. Find the rate at which the length of the edge of the cube is increasing when the volume of the cube is 64 cm^3.

(5)

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8.

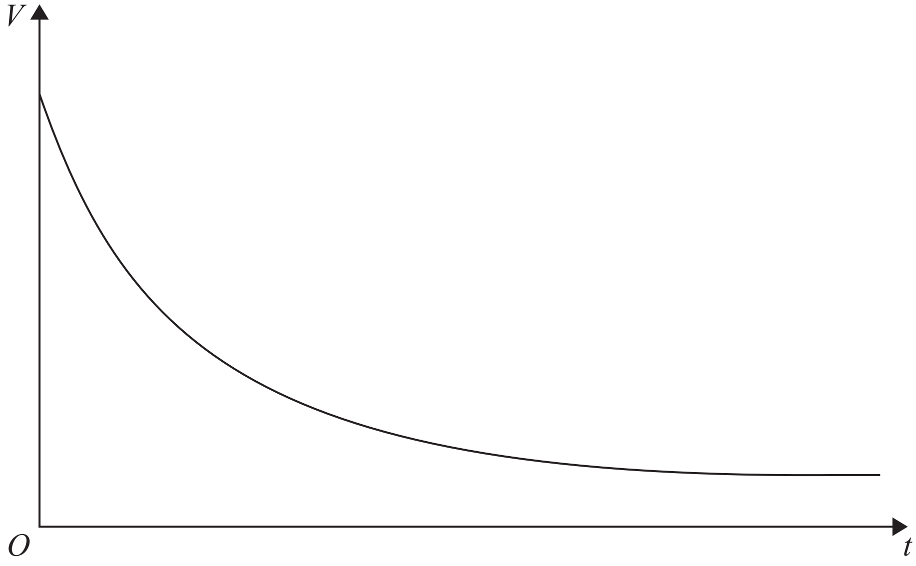

Figure 1

The value of Lin's car is modelled by the formula

$$V = 18\,000\mathrm{e}^{-0.2t} + 4000\mathrm{e}^{-0.1t} + 1000, \quad t \geqslant 0$$

where the value of the car is V pounds when the age of the car is t years.

A sketch of t against V is shown in Figure 1.

(a) State the range of V.

(2)

According to this model,

(b) find the rate at which the value of the car is decreasing when $t = 10$
Give your answer in pounds per year.

(3)

(c) Calculate the exact value of t when $V = 15000$

(4)

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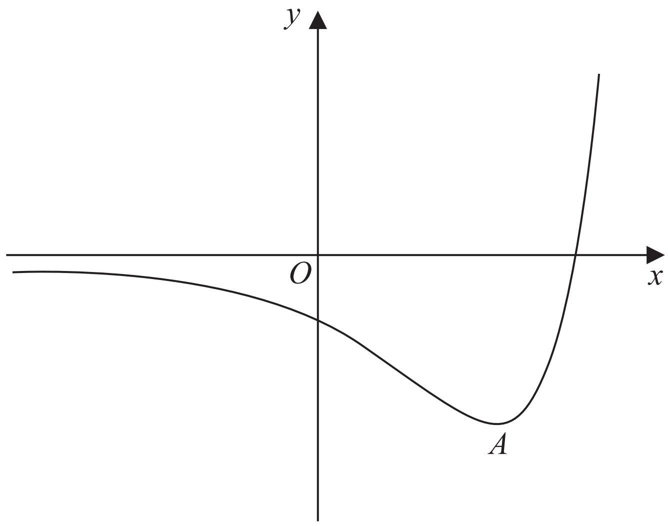

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x - 5)e^x, \qquad x \in \mathbb{R}$$

The curve has a minimum turning point at A.

(a) Use calculus to find the exact coordinates of A.

(5)

Given that the equation $f(x) = k$, where k is a constant, has exactly two roots,

(b) state the range of possible values of k.

(2)

(c) Sketch the curve with equation $y = |f(x)|$.

Indicate clearly on your sketch the coordinates of the points at which the curve crosses or meets the axes.

(3)

Leave blank

4. $$g(x) = \frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12}, \qquad x > 3, \quad x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \equiv x^2 + A + \frac{B}{x - 3}$$

find the values of the constants A and B.

(4)

(b) Hence, or otherwise, find the equation of the tangent to the curve with equation $y = g(x)$ at the point where $x = 4$. Give your answer in the form $y = mx + c$, where m and c are constants to be determined.

(5)

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6. (i) Differentiate $y = 5x^2 \ln 3x, \quad x > 0$

(2)

(ii) Given that

$$y = \frac{x}{\sin x + \cos x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

(4)

9.

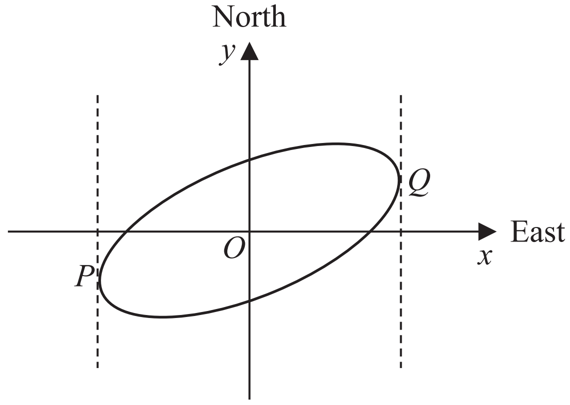

Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{dy}{dx} = \dfrac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

15. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

Leave blank

5. (a) Prove, by using logarithms, that

$$\frac{d}{dx}(2^x) = 2^x \ln 2$$

(3)

The curve C has the equation

$$2x + 3y^2 + 3x^2y + 12 = 4 \times 2^x$$

The point P, with coordinates (2, 0), lies on C.

(b) Find an equation of the tangent to C at P.

(6)

Leave blank

2. A curve C has the equation

$$x^3 - 3xy - x + y^3 - 11 = 0$$

Find an equation of the tangent to C at the point $(2, -1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Leave blank

1. A curve has equation

$$4x^2 - y^2 + 2xy + 5 = 0$$

The points P and Q lie on the curve.

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ at P and at Q,

(a) use implicit differentiation to show that $y - 6x = 0$ at P and at Q.

(6)

(b) Hence find the coordinates of P and Q.

(3)

Leave blank

3. A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (2, 3). Give your answer in the form $\frac{a + \ln b}{8}$, where a and b are integers.

(7)

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2. The point P with coordinates $\left(\frac{\pi}{2}, 1\right)$ lies on the curve with equation

$$4x\sin x = \pi y^2 + 2x, \qquad \frac{\pi}{6} \leqslant x \leqslant \frac{5\pi}{6}$$

Find an equation of the normal to the curve at P.

(6)

Leave blank

1. Find an equation of the tangent to the curve

$$x^3 + 3x^2y + y^3 = 37$$

at the point (1, 3). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Leave blank

1. A curve C has equation

$$3x^2 + 2xy - 2y^2 + 4 = 0$$

Find an equation for the tangent to C at the point (2, 4), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

Leave blank

2. The curve C has equation

$$y^3 + x^2y - 6x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence find the exact coordinates of the points on C for which $\frac{dy}{dx} = 0$

(6)

Leave blank

4. The curve C has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point P with coordinates $(-2, 4)$ lies on C.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

(6)

The normal to C at P meets the y-axis at the point A.

(b) Find the y coordinate of A, giving your answer in the form $p + q\ln 2$, where p and q are constants to be determined.

(3)

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3. The curve C has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P with coordinates $\left(3, \frac{1}{2}\right)$ lies on C.

The normal to C at P meets the x-axis at the point A.

(b) Find the x coordinate of A, giving your answer in the form $\frac{a\pi + b}{c\pi + d}$, where a, b, c and d are integers to be determined.

(4)

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2. The curve C has equation

$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that $\frac{dy}{dx}$ at the point (1, 3) on the curve C can be written in the form $\frac{1}{\lambda}\ln(\mu e^3)$, where λ and μ are integers to be found.

(7)

13.

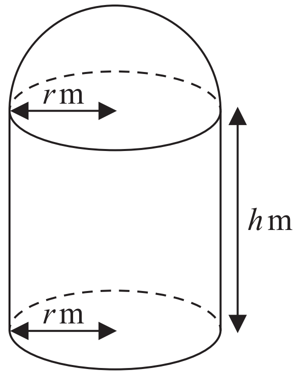

Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is $6\,\text{m}^3$.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

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14.

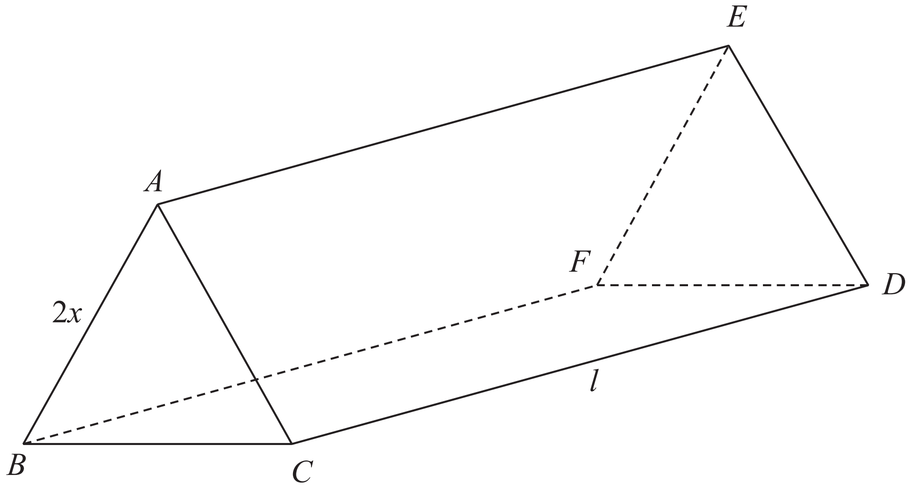

Figure 6

Figure 6 shows a solid triangular prism *ABCDEF* in which $AB = 2x$ cm and $CD = l$ cm.

The cross section *ABC* is an equilateral triangle.

The rectangle *BCDF* is horizontal and the triangles *ABC* and *DEF* are vertical.

The total surface area of the prism is S cm^2 and the volume of the prism is V cm^3.

(a) Show that $S = 2x^2\sqrt{3} + 6xl$

(3)

Given that $S = 960$,

(b) show that $V = 160x\sqrt{3} - x^3$

(5)

(c) Use calculus to find the maximum value of V, giving your answer to the nearest integer.

(5)

(d) Justify that the value of V found in part (c) is a maximum.

(2)

Leave blank

16. [*In this question you may assume the formula for the area of a circle and the following formulae:*

*a **sphere** of radius r has volume* $V = \frac{4}{3}\pi r^3$ *and surface area* $S = 4\pi r^2$

*a **cylinder** of radius r and height h has volume* $V = \pi r^2 h$ *and curved surface area* $S = 2\pi rh$]

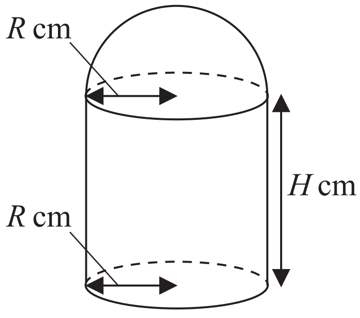

Figure 5

Figure 5 shows the model for a building. The model is made up of three parts. The roof is modelled by the curved surface of a hemisphere of radius R cm. The walls are modelled by the curved surface of a circular cylinder of radius R cm and height H cm. The floor is modelled by a circular disc of radius R cm. The model is made of material of negligible thickness, and the walls are perpendicular to the base.

It is given that the volume of the model is 800π cm^3 and that $0 < R < 10.6$

(a) Show that

$$H = \frac{800}{R^2} - \frac{2}{3}R$$

(2)

(b) Show that the surface area, A cm^2, of the model is given by

$$A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$$

(3)

(c) Use calculus to find the value of R, to 3 significant figures, for which A is a minimum.

(5)

(d) Prove that this value of R gives a minimum value for A.

(2)

(e) Find, to 3 significant figures, the value of H which corresponds to this value for R.

(1)

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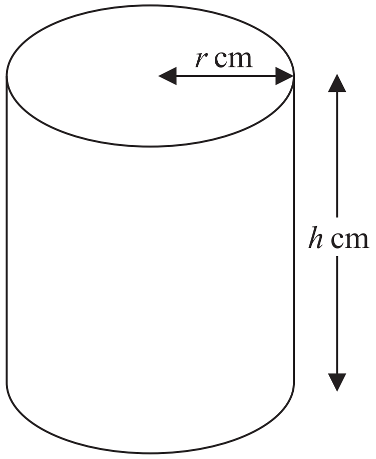

Figure 5

Figure 5 shows a design for a water barrel.

It is in the shape of a right circular cylinder with height *h* cm and radius *r* cm.

The barrel has a base but has no lid, is open at the top and is made of material of negligible thickness.

The barrel is designed to hold 60 000 cm^3 of water when full.

(a) Show that the total external surface area, *S* cm^2, of the barrel is given by the formula

$$S = \pi r^2 + \frac{120\,000}{r}$$

(3)

(b) Use calculus to find the minimum value of *S*, giving your answer to 3 significant figures.

(6)

(c) Justify that the value of *S* you found in part (b) is a minimum.

(2)

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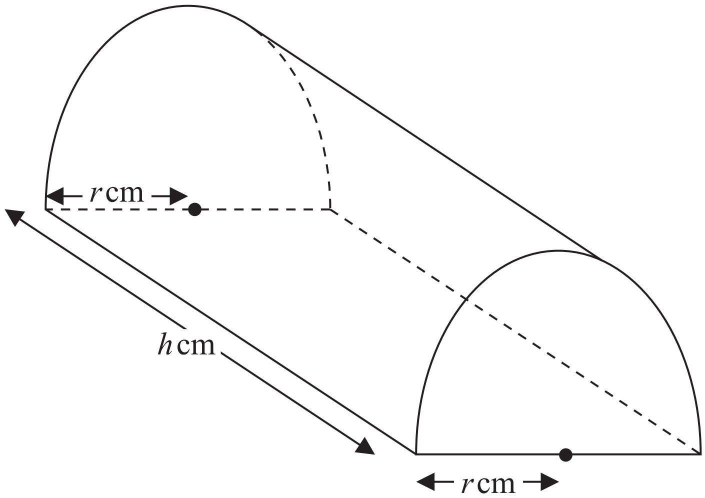

Figure 4

Figure 4 shows a solid wooden block. The block is a right prism with length h cm. The cross-section of the block is a semi-circle with radius r cm.

The total surface area of the block, including the curved surface, the two semi-circular ends and the rectangular base, is 200 cm^2

(a) Show that the volume V cm^3 of the block is given by

$$V = \frac{\pi r(200 - \pi r^2)}{4 + 2\pi}$$

(5)

(b) Use calculus to find the maximum value of V. Give your answer to the nearest cm^3.
(6)

(c) Justify, by further differentiation, that the value of V that you have found is a maximum.

(2)

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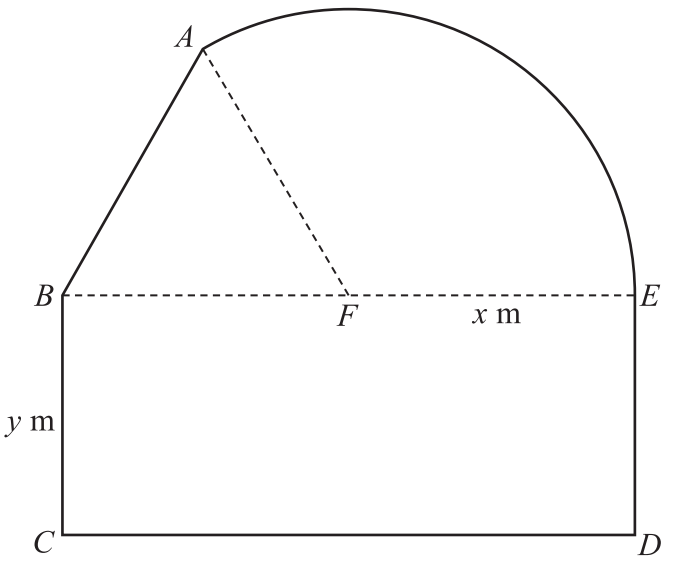

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$

(a) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form.
(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is 1000 m^2,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$$

(3)

(c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$$

(3)

(d) Use calculus to find the minimum value of P, giving your answer to the nearest metre.
(5)

(e) Justify, by further differentiation, that the value of P you have found is a minimum.
(2)

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9. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

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10.

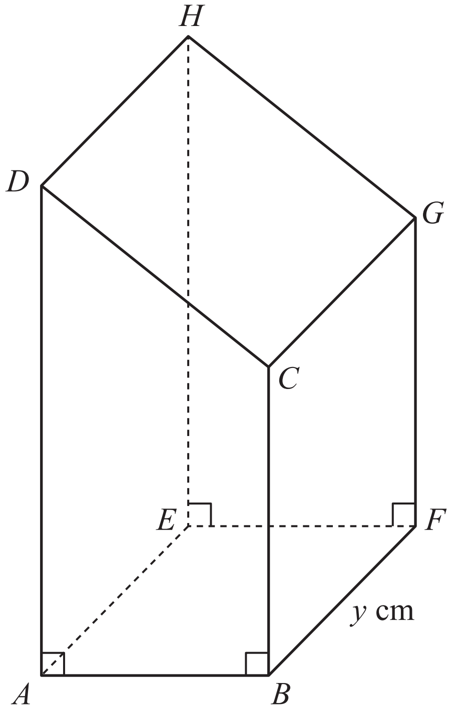

Figure 4

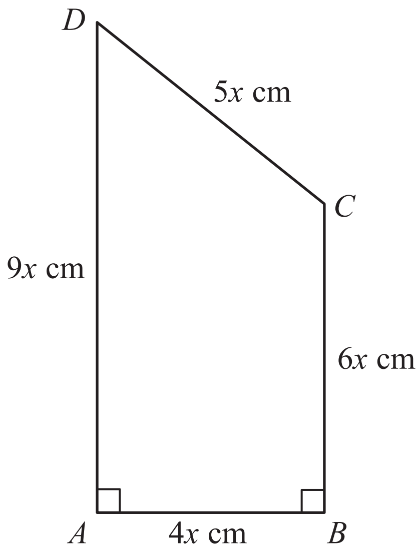

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD,* which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is $S\,\text{cm}^2$.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.
The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is 9600 cm^3.

(a) Show that

$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, $S\,\text{cm}^2$, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

P 4 3 1 7 7 A 0 3 2 3 6

Leave blank

8.

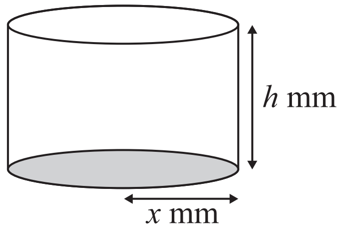

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x,

(1)

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum.

(5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(e) Show that this value of A is a minimum.

(2)

Leave blank

8.

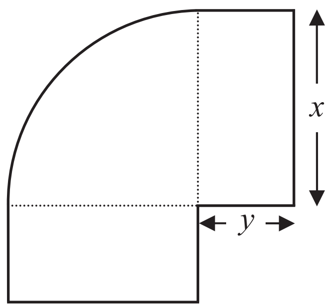

Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2,

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)